Pure Mathematics 30

Student Project: Winter Sports Competitions

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Alberta Education

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Project—Winter Sports Competitions

Introduction

In any sports competition, mathematics plays a major role. That role may include training of athletes, the design of the playing area, the measuring of the athletes' performances, and the scoring.

Part A

An international curling competition has 10 teams. In the round-robin portion of the competition, each team plays every other team once; the top four teams then qualify for the medal round.

1. Determine the number of games that are played during the round-robin portion of the competition.

2. In the medal round, there are two semifinal games. In one, the first-place team plays the fourth-place team, and in the other, the second-place team plays the third-place team. Determine the number of different possible pairings for the semifinal games. Explain your answer.

Organizers propose expanding the competition to a total of 12 teams, using one of two methods:

Method A: Have a 12-team round robin, with the top four teams qualifying for the medal round

Method B: Have 2 groups of 6 teams each with its own round robin, with the top two teams in each group qualifying for the medal round

3. For each method, determine the number of games played during the round-robin portion of the competition.

4. Identify the factors that need to be considered in order to decide which method should be selected.

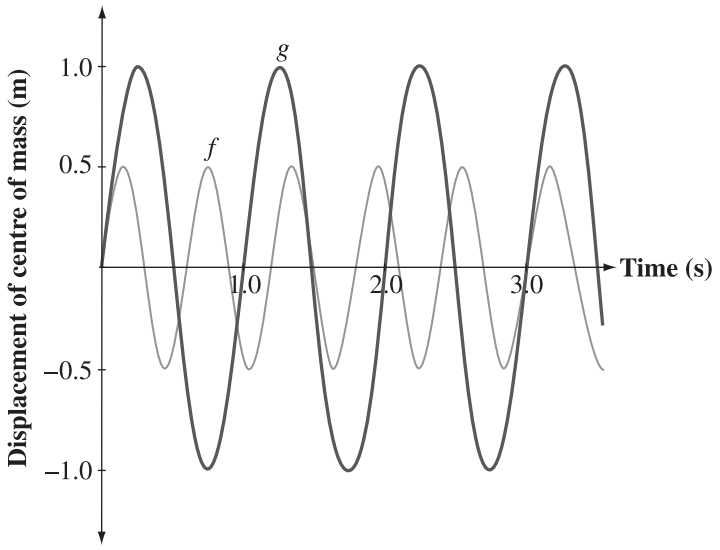
5. If there are no restrictions, determine the number of ways of splitting the 12 teams into 2 groups of 6 teams. State any assumptions made in determining your solution.

With Method B there is a risk that all the strong teams could be in one of the two groups, making it potentially unfair for the strongest teams. To reduce this risk, organizers rank the 4 strongest teams in order from first to fourth. One group will have the teams ranked first and fourth, and the other group will have the teams ranked second and third. The other 8 teams are allocated to the two groups in a random manner.

6. With these restrictions, determine the number of ways of splitting the 12 teams into 2 groups of 6 teams. State any assumptions made in determining your solution.

Part B

When a speed skater is striding, her centre of mass is displaced side to side with respect to her direction of travel. When she is maintaining a constant speed, this displacement is greater than when she is accelerating. The displacement of a speed skater's centre of mass can be approximately modelled using a sinusoidal function. The displacement of this particular speed skater's centre of mass, in metres, as a function of time, in seconds, during two particular portions of a race is partially graphed below.



1. • State the period of each function.

• During which part of the race, beginning or middle, would strides be found that are modelled by each function? Justify your answer with references to the periods of the functions.

2. The graph of $y = f(t)$ models the displacement of the speed skater's centre of mass during acceleration as a function of time. Determine the function in the form $f(t) = a \cdot \sin\left[b(t - c)\right] + d$.

3. The graph of $y = g(t)$ models the displacement of the speed skater's centre of mass during a portion of a race in which she is maintaining a constant speed. Determine the function $g(t)$ in the form $g(t) = a \cdot \sin\left[b(t - c)\right] + d$.

4. Describe the transformations that are required to transform $g(t)$ to $f(t)$.

Competitive skiers are frequently tested for their red blood cell count. In a normal population, red blood cell count levels typically range from 4 400 cells/nL to 5 800 cells/nL for males, and 4 000 cells/nL to 5 200 cells/nL for females. These figures are based on the mean ± 2 standard deviations for each gender.

5. Since 5 800 cells/nL and 5 200 cells/nL both represent a z-score of $+2$ for the genders, determine the percentage of any population to the right of 2 standard deviations under a standard normal curve.

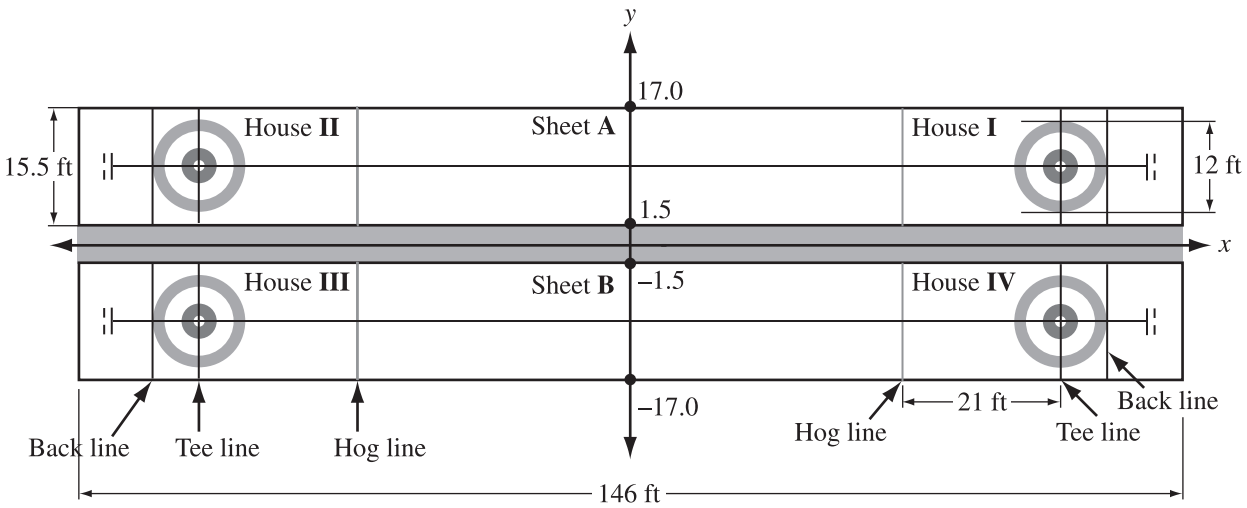
Part C

Many winter sports competitions feature curling.

Two adjacent curling sheets, A and B, are shown in the diagram below. Each house, or target area, contains 4 concentric circles of the following radii: 0.5 ft, 2 ft, 4 ft, and 6 ft. The areas between the concentric circles are shaded, as shown below.

Each sheet has a length of 146 ft and a width of 15.5 ft. Each sheet has 2 hog lines that are 72 ft apart and equidistant from the corresponding back line.

A Cartesian coordinate system is marked on the diagram below.



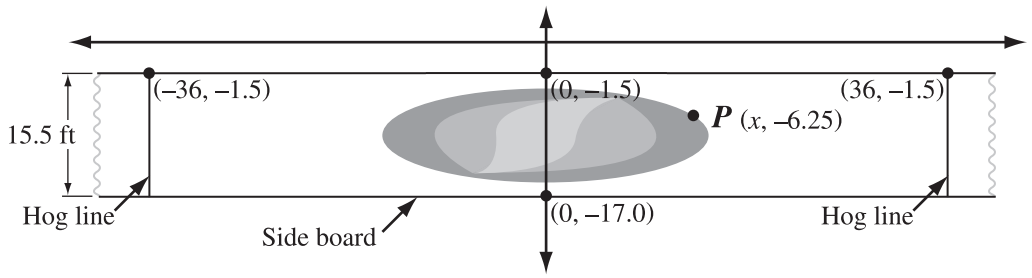
1. • State the equation of the outermost circle in House I in the standard form $(x - h)^2 + (y - k)^2 = r^2$.

• Referring to lines parallel to the x-axis and y-axis, describe a set of transformations that would transform the innermost circle of House I to the outermost circle of House I.

2. • Describe a set of transformations that would transform the innermost circle in House I to the innermost circle in House III.

• Describe a **different** set of transformations that would transform the innermost circle in House I to the innermost circle in House III.

The organizers of one competition are considering placing an elliptical logo on sheet B with the longer axis of the ellipse parallel to the side board. The centre of the logo is at the centre of sheet B. The boundary of the ellipse must be 2.75 ft inside each side board and 16 ft from each hog line, as shown below.



3.
- State the coordinates of the centre of the elliptical logo.
- Determine the equation that represents the outer edge of this logo in standard form.
- Determine the equation that represents the outer edge of this logo in general form.

4. When a rock is thrown (slid down the ice), curling rules stipulate that it must stop past the hog line. The front edge of a particular rock that has been thrown lies at point $P(x, -6.25)$ on the outer edge of the logo, as shown in the diagram above. How many feet short of the hog line is the front edge of this rock?

The area of an ellipse is given by $A = \pi ab$, where a and b are the parameters in the standard form equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

5.
- Determine the area, to the nearest hundredth of a square foot, of the elliptical logo on sheet B.
- Determine the area of sheet B that is **not** enclosed in a circle or an ellipse.

The ideal curling ice-surface temperature is between $-5.0\ °C$ and $-4.5\ °C$. Curling ice-surface temperatures for a particular venue are normally distributed.

Prior to the start of a competition, officials test the curling ice-surfaces and find that the mean temperature is $-4.75\ °C$.

6. Determine the largest standard deviation of the ice-surface temperatures, to the nearest thousandth, that would allow 80% of these ice-surface temperatures to be within the ideal range described above.

Part D

1. Research and write a paper on

- the changes in technology in international curling,

or

- comparing and contrasting speed skating on short and long tracks,

or

- the Sandra Schmirler Foundation.

2. Research the Internet for information on red blood cell count levels in elite athletes by entering "red blood cell counts in elite athletes" in your favourite search engine. These levels can be affected by such factors as training regimes, altitude, or performance-enhancing substances like erythropoietin (EPO).

The following websites may help in your research:

www.curling.ca

www.speedskating.ca

www.canada2010.gc.ca

www.vancouver2010.com

Note: Website addresses sometimes change. If the websites listed above are not available, use a search engine and type in keywords such as those listed below:

"curling"

"speed skating"