# Information Bulletin | Pure Mathematics 30

## 2011 – 2012 Diploma Examinations Program

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This document was written primarily for:

| | |
|---|---|
| Students | ✓ |
| Teachers | ✓ of Pure Mathematics 30 |
| Administrators | ✓ |
| Parents | |
| General Audience | |
| Others | |

Distribution: This document is posted on the Alberta Education website at education.alberta.ca.

# *Contents*



Please note that if you cannot access one of the direct website links referred to in this document, you can find Diploma Examination-related materials on the Alberta Education website at education.alberta.ca.

At the home page, click on the link *Provincial Testing* under *For Administrators*. Next click on *Diploma Examinations* and then one of the specific links listed under the *Diploma Examinations* heading.

# Introduction

The purpose of this bulletin is to provide students and teachers of Pure Mathematics 30 with information about the diploma examinations scheduled in the 2011–2012 school year. This bulletin should be used in conjunction with the current *Pure Mathematics 30 Program of Studies* to ensure that the curriculum and standards are addressed.

This bulletin includes descriptions of the Pure Mathematics 30 Diploma Examinations that will be administered in November 2011 and in January, June, and August 2012, descriptions of the *acceptable standard* and the *standard of excellence*, and subject-specific information. The mark awarded to a student on the Pure Mathematics 30 Diploma Examinations in the 2011–2012 school year will account for 50% of the student's final blended mark, and the school-awarded mark will account for the remaining 50%.

For students with mature-student status, the mark awarded on the Pure Mathematics 30 Diploma Examination will account for 100% of the final blended mark if the examination mark is higher than the school-awarded mark.

Teachers are encouraged to share the contents of this bulletin with students.

For further information about program implementation, refer to the Alberta Education website at education.alberta.ca.

# Course Objectives

The Pure Mathematics 30 course emphasizes mathematical theory. In pure mathematics, algebraic and graphical approaches are used to solve problems. Deductive and symbolic methods are used to determine whether and under what conditions a concept is true.

Students are expected to communicate solutions to problems clearly and effectively when solving both routine and non-routine problems. Technology is to be used for exploration, modelling, and problem solving. Students are also expected to apply mathematical concepts and procedures to real-life problems.

# *Teacher Involvement in the Diploma Examination Process*

High-quality diploma examinations are the product of close collaboration between classroom teachers and Alberta Education. Classroom teachers from across Alberta are involved in many aspects of diploma examination development, including the development of raw items; the building, reviewing, and administering of field tests; and the reviewing of diploma examination drafts.

Alberta Education values the involvement of teachers and often asks school jurisdictions for the names of teachers who are interested in being involved. Teachers who are interested in developing raw items or building and/or reviewing field tests are encouraged to ask their principals to submit their names, through proper channels, to the Assessment Sector. The list of teachers interested in these aspects of the development process remains open all year long, and teachers are welcome to have their names submitted at any time.

Other opportunities to be involved, such as field testing, have specific closing dates. General dates to be aware of include:

| | |
|---|---|
| September 2011 | Registration for field tests to be administered in December 2011 or January 2012 |
| February 2012 | Registration for field tests to be administered in May or June 2012 |

*NEW*

# *Online Field Testing*

Field tests for Pure Mathematics 30 in the 2011–2012 school year will be offered in digital format only. Partial year-end field tests will be available which consist of the following units:

Transformations of Functions

Exponents, Logarithms, and Geometric Series

Trigonometry

Permutations and Combinations

These field tests will **not** include Conic Sections and Statistics and they will cover only those outcomes from Pure Mathematics 30 that will carry over into Mathematics 30–1. Field test request procedures can be found in the *Diploma General Information Bulletin* on the Alberta Education website at education.alberta.ca by following the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Diploma General Information Bulletin*.

# *Performance Expectations*

### Curriculum Standards

Provincial curriculum standards help to communicate how well students need to perform in order to be judged as having achieved the learning outcomes specified for Pure Mathematics 30. The specific statements of standards are written primarily to apprise Pure Mathematics 30 teachers of the extent to which students must know the Pure Mathematics 30 curriculum and demonstrate the required skills in order to pass the examination.

# *Performance Standards*

### Acceptable Standard

Students who attain the *acceptable standard* but not the *standard of excellence* in Pure Mathematics 30 will receive a final course mark between 50% and 79%, inclusive. Typically, these students have gained new skills and knowledge in mathematics and can apply mathematical concepts and procedures to find a solution to routine problems. They have demonstrated mathematical skills and knowledge in the six units of the Pure Mathematics 30 curriculum and have exhibited an ability to apply a broad range of problem-solving skills to these units.

### Standard of Excellence

Students who attain the *standard of excellence* will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and have confidence in their mathematical abilities. These students can choose the most efficient method for solving problems. They can also find more than one solution and can solve non-routine problems.

# *Examples of Questions*

This bulletin contains an appendix with an example of a written-response question, sample student responses, and scoring rationales as they relate to the general scoring guide. For more examples of questions, please refer to the *Pure Mathematics 30 Archived Information Bulletin*.

## *Projects*

Teachers are encouraged to access past projects that can be found on the Alberta Education website at education.alberta.ca by following the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Projects for Pure and Applied Mathematics 30*. These projects are designed to be completed in three to five hours of student time. Use of these projects is optional. A sample solution and a scoring guide have been provided for each project, and teachers may choose to use the projects as part of their assessment.

### *NEW*

## *Format Changes*

The formatting of content in some examination booklets has changed slightly. The instructions pages now begin on the inside front cover, and the side, top, and bottom page margins are narrower than before. **The changes are not a misprint.** As a result of these changes, the total amount of paper used each year in printing the examinations will decrease by several tonnes.

The format changes do not apply to all diploma examination booklets. French-language booklets, Part A booklets, and Readings booklets still use the old format. Also, the size of the print is unchanged.

## *Examination Specifications and Design*

Each Pure Mathematics 30 Diploma Examination is designed to reflect the core content outlined in the *Pure Mathematics 30 Program of Studies*. The examination is limited to those expectations that can be measured by a machine-scored paper-and-pencil test. Therefore, the percentage weightings will not necessarily match the percentage of class time devoted to each unit. The Pure Mathematics 30 diploma examination will be two hours, with an additional half hour if needed.

**Specifications**

The question format of the Pure Mathematics 30 Diploma Examinations in the 2011–2012 school year is as follows:

| *Question Format* | *Number of Questions* | *Percentage Emphasis* |
|---|---|---|
| Multiple Choice | 33 | 82 |
| Numerical Response | 7 | 18 |

The content of the Pure Mathematics 30 Diploma Examinations in the 2011–2012 school year is emphasized as follows:

| ***Diploma Exam Content*** | ***Percentage Emphasis*** |
|---|---|
| Transformations of Functions | 15 |
| Exponents, Logarithms, and Geometric Series | 20 |
| Trigonometry | 24 |
| Conic Sections | 12 |
| Permutations and Combinations | 19 |
| Statistics | 10 |

**Machine-Scored Questions**

Information required to answer **multiple-choice** and/or **numerical-response questions** is often located in a box preceding the question. The number of questions that require the use of the information given in the box will be clearly stated above the box; e.g., "*Use the following information to answer the next two questions*."

For **multiple-choice questions**, students are to choose the correct or best possible answer from four alternatives.

The **numerical-response questions** are interspersed throughout the multiple-choice questions, according to content topic.

For some numerical-response questions, students are required to calculate a numerical answer and then record their answer in a separate area of the answer sheet. When the answer to be recorded cannot be a decimal value, students are asked to determine a whole number value (e.g., the number of people is ______; the number of different routes is ______). If the answer can be a decimal value, then students are asked to record their answer to the nearest tenth or nearest hundredth, as specified in the question. Students should retain all decimals throughout the question and **rounding should occur only in the final answer**.

Other numerical-response questions require students to record their understanding of a concept. The following is an example of such a question.

*Use the following information to answer the next question.*

| | |
|---|---|
| The three expressions below each have a particular numerical value. | |
| **1** | $_{7}C_{2}$ |
| **2** | $_{5}P_{2}$ |
| **3** | $4!$ |

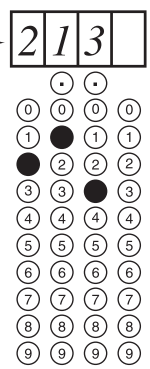
**Numerical Response**

**1.** If the expressions above are ordered from lowest numerical value to highest numerical value, then their corresponding expression numbers will be _____, _____, and _____.

(Record all **three digits** of your answer in the numerical-response section on the answer sheet.)

**Answer: 213**

**Record 213 on the answer sheet** →




# *Using Calculators*

The Pure Mathematics 30 Diploma Examination requires the use of an approved graphing calculator. The calculator directives, expectations, criteria, and keystrokes required for clearing approved calculators can be found in the *General Information Bulletin* on the Alberta Education website at education.alberta.ca by following the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Diploma General Information Bulletin > Using Calculators & Computers*.

# *Examination Security*

All Pure Mathematics 30 Diploma Examinations will be held secure until released to the public by the Minister. However, for the January and June 2012 examinations, teachers will be allowed access to a *Teacher Perusal Copy* for review purposes one hour after the examination has started. Portions of previous diploma examinations administered in 2010 or earlier will be released in the fall of 2011.

# *Publications and Supporting Documents*

The following documents are produced to provide teachers with information about the Pure Mathematics 30 Diploma Examination:

*Pure Mathematics 30 Released Items* available at education.alberta.ca, via the pathway *Teachers > (Additional Programs and Services) Diploma Exams > Released Materials*

*Pure Mathematics 30 Assessment Highlights* available at education.alberta.ca, via the pathway *Teachers > (Additional Programs and Services ) Diploma Exams > Assessment Highlights*

*Pure Mathematics 30 Information Bulletin* available at education.alberta.ca, via the pathway *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins*

*School Reports and Instructional Group Reports* for January and June Diploma Examinations available on the extranet at https://phoenix.edc.gov.ab.ca/login/default.asp

# *Maintaining Consistent Standards Over Time on Diploma Examinations*

A goal of Alberta Education is to make examinations directly comparable from session to session, thereby enhancing fairness to students across administrations.

To achieve this goal, a number of questions, called anchor items, remain the same from one examination to another. Anchor items are used to find out whether the student population writing in one administration differs in achievement from the student population writing in another administration. Anchor items are also used to find out whether the unique items (questions that are different on each examination) differ in difficulty from the unique items on the baseline examination (the first examination to use anchor items). A statistical process, called equating or linking, adjusts for the differences in examination-form difficulty. Examination marks may be adjusted slightly upward or downward, depending upon the difficulty of the examination written relative to the baseline examination. The resulting equated or linked examination scores have the same meaning regardless of when and to whom the examination was administered. Equated or linked diploma examination marks will be reported to students.

Because of the security required to enable fair and appropriate assessment of student achievement over time, diploma examinations may have to be fully secured on occasion and will not be released at the time of writing. Please check the *General Information Bulletin* (available at education.alberta.ca, via the pathway: *Teachers* > (*Additional Programs and Services*) *Diploma Exams* > *Diploma General Information Bulletin* or the information bulletins for each diploma subject you teach to determine which examinations are fully secured. For more information about equating or linking, please go to the Alberta Education website at education.alberta.ca, via the pathway: *Teachers* > (*Additional Programs and Services*) *Diploma Exams* > *Initiative to Maintain Consistent Standards on Diploma Examinations*.

# *Explanation of Cognitive Levels*

Procedural, conceptual, and problem-solving cognitive levels are addressed throughout the examination. The emphasis on each cognitive level will be approximately equal.

**Procedural**

The assessment of students' knowledge of mathematical procedures should involve recognition, defence, execution, and verification of appropriate procedures and the steps contained within them. The use of technology can allow for conceptual understanding prior to specific skill development or vice versa. Students must appreciate that procedures are created or generated to meet specific needs in an efficient manner and thus can be modified or extended to fit new situations. Assessment of students' procedural knowledge will not be limited to an evaluation of their proficiency in performing procedures, but will be extended to reflect the skills presented above.

To ensure fairness to all students regardless of their choice of calculators, certain types of procedural execution will not be tested on diploma examinations. This procedural execution is, however, an integral part of the program of studies and should be tested in the classroom.

**Conceptual**

An understanding of mathematical concepts goes beyond a mere recall of definitions and recognition of common examples. Assessment of students' knowledge and understanding of mathematical concepts should provide evidence that they can compare, contrast, label, verbalize and define concepts, identify and generate examples and counter-examples as well as properties of a given concept, and recognize the various meanings and interpretations of concepts. Students who have developed a conceptual understanding of mathematics can also use models, symbols, and diagrams to represent concepts. Appropriate assessment will also provide evidence of the extent to which students have integrated their knowledge of various concepts.

**Problem Solving**

Appropriate assessment of problem-solving skills is achieved by allowing students to adapt and extend the mathematics they know and by encouraging the use of strategies to solve unique and unfamiliar problems. Assessment of problem solving involves measuring the extent to which students use these strategies and knowledge, and their ability to verify and interpret results. Students' ability to solve problems develops over time as a result of their experiences with relevant situations that present opportunities to solve various types of problems.

Evidence of problem-solving skills is often linked to clarity of communication. Students demonstrating strong problem-solving skills should be able to clearly explain the process they have chosen, using clear language and appropriate mathematical notation and conventions.

# *Mathematical Processes*

**Communication (C)**

Students need to communicate mathematical ideas clearly and effectively. In communicating answers, students must be aware of the degree of accuracy required as well as the appropriate units involved.

**Connections (CN)**

When mathematical ideas are connected to each other through concrete, pictorial and symbolic representations, students begin to view mathematics as an integrated whole. Students need numerous and varied experiences to appreciate the usefulness of mathematics. They must explore connections between different areas of mathematics and between mathematics and other disciplines. The "connections process" also includes relating mathematics to their own daily experiences.

The connections process is often linked with problem solving and reasoning because it is an application of these other processes.

**Problem Solving (PS)**

Problem solving is the focus of mathematics at all grade levels. The development of each student's ability to solve problems is essential. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Approximately one-third of the machine-scored questions will be related to a problem-solving context. Students are expected to select an appropriate problem-solving strategy to find the solution to a given problem or to model real-world problem situations.
Students should not expect questions on a particular concept to be asked in the same manner every time. They must be able to adapt to changes in the format, which will help to improve their problem-solving skills.

**Reasoning (R)**

Students need to develop confidence in their ability to reason and to justify their thinking within and outside mathematics. The power of reasoning helps students to make sense of mathematics, to be logical in their thinking, and to convince others.

Inductive reasoning helps students explore and make conjectures from activities that allow generalizations from a pattern of observations.

Deductive reasoning helps students test conjectures and build arguments that serve to validate logical thinking. Deductive reasoning also assembles a structured body of knowledge.

Reasoning allows students to interpret mathematics and aids in the choice of appropriate problem-solving strategies.  Students may be asked to demonstrate logical reasoning when judging the validity of arguments, testing conjectures, and constructing arguments.

**Technology (T)**

"Electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics.  They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately.  They can support investigation by students in every area of mathematics, including geometry, statistics, algebra, measurement, and number.  When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving….  Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions." (*Principles and Standards for School Mathematics, NCTM 2000*, pages 24 to 25).

# *Website Links*

| Publication/Resource Pure Mathematics Program of Studies | Website |
|---|---|
| Diploma General Information Bulletin | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Diploma General Information Bulletin* |
| Pure Mathematics 30 Archived Information Bulletin | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins* |
| Using Graphing Calculators | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins > Using Calculators and Computers* |
| Pure Mathematics 10, 20, and 30 Program of Studies | *education.alberta.ca*, via the pathway: *Teachers > (Programs of Study) > Mathematics > Educators > Programs of Study* |
| Quest A+ | *https://questaplus.alberta.ca* |
| Projects | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Projects* *Note: Sample solutions can be found on the extranet. |
| Mathematics and Science Directing Words | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins > Mathematics and Science Directing Words* |
| Assessment Highlights | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Assessment Highlights* |
| Released Items | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Released Materials* |
| Previously Released Diploma Examinations and Answer Keys | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Previous Diploma Examinations and Answer Keys* |
| **Publication – Revised Mathematics Program of Studies** | **Website** |
| The Revised Mathematics Program of Studies | *education.alberta.ca*, via the pathway: *Teachers > (Programs of Study) > Mathematics > Educators > Programs of Study* |
| The High School Information Package | *education.alberta.ca*, via the pathway: *Teachers > (Programs of Study) > Mathematics* |
| Using Graphing Calculators on Mathematics 30–1 and 30–2 Diploma Examinations | *education.alberta.ca*, via the pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins > Approved Graphing Calculators for Mathematics 30–1, Mathematics 30–2, and Science Diploma Examinations in 2012–2013* |

*NEW*

# *Revised Mathematics Program of Studies Implementation*

The revised *Mathematics 10–12 Program of Studies (2008)* will be implemented according to the following schedule:

September 2011: Mathematics 20–1
September 2012: Mathematics 30–1

The program of studies can be downloaded from the Alberta Education website at education.alberta.ca by following this path: *Teachers > (Programs of Study) > Mathematics > Educators > Programs of Study.*

The revised *Mathematics Grades 10–12 (2008)* is listed under the **Programs of Study** heading.

The *Mathematics 30–1 Information Bulletin 2012–2013* will contain the following information:

- Standards appropriate to Mathematics 30–1 in the revised *Mathematics 10–12 Program of Studies*, as developed by teachers from across Alberta in cooperation with Alberta Education
- Notes for teachers
- Standards statements for the acceptable standard as well as the standard of excellence
- The examination's design in terms of percentage weightings by topic, number, and type of questions, and percentage weightings by mathematical understanding
- Sample questions
- The formula sheet

A draft version of the *Mathematics 30–1 Information Bulletin 2012–2013* will be available in the spring of 2012. If you have any questions about the diploma examination program for Mathematics 30–1, please contact Ross Marian at Ross.Marian@gov.ab.ca or within Alberta toll free at 310–0000 then 780–427–0010.

### Grandfathering of Old Program of Studies

The last regular administration of the Pure Mathematics 30 Diploma Examination will be in August 2012. As this course will no longer be offered starting in the 2012–2013 school year, the Pure Mathematics 30 Diploma Examinations will be available after September 2012 **only** for those students who wish to re-write the exam or who began the course using distributed learning materials or on a Copernican timetable prior to the end of August 2012.

# *Pure Mathematics 30 Formula Sheet*

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For two points $(x_1, y_1)$ and $(x_2, y_2)$,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Exponents, Logarithms, and Geometric Series**

$A = P(1 + i)^n$

$\log_a(M \times N) = \log_a M + \log_a N$

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a M^n = n \log_a M$

$\log_b c = \frac{\log_a c}{\log_a b}$

$t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$

$S_n = \frac{(rt_n - a)}{r - 1}, \ r \neq 1$

**Statistics and Probability**

$z = \frac{x - \mu}{\sigma}$

$P(k) = {}_nC_k p^k (1 - p)^{n-k}$

**Permutations and Combinations**

${}_nP_r = \frac{n!}{(n - r)!}$

${}_nC_r = \frac{n!}{(n - r)!r!}$

In the expansion of $(x + y)^n$,
the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$.

$n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$, where
$n \in N$ and $0! = 1$

Graphing Calculator Window Format

$x$: $[x_{\text{min}},\ x_{\text{max}},\ x_{\text{scl}}]$

$y$: $[y_{\text{min}},\ y_{\text{max}},\ y_{\text{scl}}]$

**Trigonometry**

$a = r\theta$

$\csc x = \frac{1}{\sin x}$ $\qquad$ $\sec x = \frac{1}{\cos x}$

$\cot x = \frac{1}{\tan x}$

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\sin(2A) = 2 \sin A \cos A$

$\cos(2A) = \cos^2 A - \sin^2 A$

**Conics**

*General Form*

$Ax^2 + Cy^2 + Dx + Ey + F = 0$
where $A, C, D, E, F \in I$

*Standard Form*

$(x - h)^2 + (y - k)^2 = r^2$

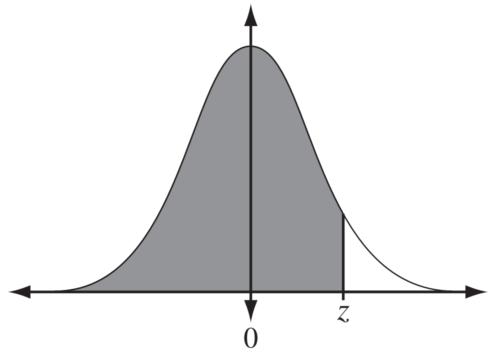
$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = \pm 1$

$y - k = a(x - h)^2$

$x - h = a(y - k)^2$

$$z = \frac{x - \mu}{\sigma}$$




***Areas Under the Standard Normal Curve***

| $z$ | **0.09** | **0.08** | **0.07** | **0.06** | **0.05** | **0.04** | **0.03** | **0.02** | **0.01** | **0.00** |
|---|---|---|---|---|---|---|---|---|---|---|
| **−3.4** | 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| **−3.3** | 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 |
| **−3.2** | 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 |
| **−3.1** | 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0010 |
| **−3.0** | 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 |
| **−2.9** | 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 |
| **−2.8** | 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 |
| **−2.7** | 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 |
| **−2.6** | 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 |
| **−2.5** | 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 |
| **−2.4** | 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 |
| **−2.3** | 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 |
| **−2.2** | 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 |
| **−2.1** | 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 |
| **−2.0** | 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 |
| **−1.9** | 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 |
| **−1.8** | 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 |
| **−1.7** | 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 |
| **−1.6** | 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 |
| **−1.5** | 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 |
| **−1.4** | 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 |
| **−1.3** | 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 |
| **−1.2** | 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 |
| **−1.1** | 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 |
| **−1.0** | 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 |
| **−0.9** | 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 |
| **−0.8** | 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 |
| **−0.7** | 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 |
| **−0.6** | 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 |
| **−0.5** | 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 |
| **−0.4** | 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 |
| **−0.3** | 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 |
| **−0.2** | 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 |
| **−0.1** | 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 |
| **−0.0** | 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 |

***Areas Under the Standard Normal Curve***

| *z* | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

# *Appendix: Other Assessment Samples for Classroom Use*

## *Introduction*

Open-ended questions provide a way to assess mathematics as a common human activity. They allow students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

Although written-response questions are no longer part of the Pure Mathematics 30 Diploma Examination, written-response and open-ended questions should be used in classroom assessment. This enables a broad-based assessment of all the outcomes in the program of studies.

In scoring the written-response questions, teachers can evaluate how well students

- understand the problem or the mathematical concept
- correctly apply mathematical knowledge and skills
- use problem-solving strategies and explain their final answers and solution procedures
- communicate their solutions and mathematical ideas

This section provides a sample written-response question, possible solutions, sample student responses, and rationales as they relate to the *general scoring guide*. These examples are intended to inform teachers and students of how the scoring guide can be applied to specific questions and to encourage the use of the *General Scoring Guide* in classroom assignments.

## *General Scoring Guide*

Student responses to written-response questions were previously scored against specific question-scoring rubrics based on the *General Scoring Guide*. Credit was given to students who appropriately demonstrated unusual insight in addressing the question.

*A “five” need not be a perfect paper!*

This scoring guide reflects a mark based on four dimensions:

- mathematical understanding
- application of processes
- clarity of communication
- use of technology

| General Scoring Guide | |
|---|---|
| **1 mark** | In the response, the student<br>• applies some relevant mathematical knowledge to explore the initial stages of the problem; however, the response reflects a misunderstanding of the problem<br>• uses a relevant strategy, mathematical process, or problem-solving technique to explore the initial stages of the problem<br>• communicates very little relevant information and lacks clarity<br>• uses technology inappropriately or the use of technology is not evident |
| **2 marks** | In the response, the student<br>• applies some relevant mathematical knowledge to find partial solutions to the problem; however, the response reflects a minimal understanding of the problem<br>• uses relevant strategies, mathematical processes, or problem-solving techniques to find a partial solution to the problem<br>• communicates strategies in a manner that lacks clarity or is incomplete<br>• uses technology where appropriate; however, errors are evident |
| **3 marks** | In the response, the student<br>• applies mathematical knowledge to find partial solutions to the problem and reflects a basic understanding of the problem<br>• uses appropriate strategies, mathematical processes, and problem-solving techniques to find partial solutions to the problem<br>• communicates the strategies and solutions in an organized manner; however, errors, inconsistencies, and omissions affect clarity<br>• uses technology appropriately; however, there are inconsistencies in their application |
| **4 marks** | In the response, the student<br>• applies appropriate mathematical knowledge to find a complete solution to the problem and reflects a good understanding of the problem<br>• uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete solution to the problem; however, the solution contains an error that hinders understanding of the response<br>• communicates strategies and solutions in an organized manner; however, errors or omissions may affect clarity<br>• uses technology appropriately |
| **5 marks** | In the response, the student<br>• applies appropriate mathematical knowledge to find a complete and correct solution to the problem and reflects an excellent understanding of the problem<br>• uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete, correct solution; the solution may have a minor error, but it does not hinder the understanding of the response<br>• communicates strategies and solutions in a clear, complete, and organized manner that reflects a thorough understanding of the problem<br>• uses technology effectively |

## *Sample Written-Response Question*

*Use the following information to answer the next question.*

The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—5 marks**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | |
| 4 | | |

*Use the following additional information to answer the next part of the question.*

When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

• **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

*Use the following additional information to answer the next part of the question.*

A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\ 500(0.97)^n$, $n \in N$.

- **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

- **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include
  - – the function or functions that would be graphed
  - – an appropriate window setting
  - – a summary of how you would use the graph or graphs to determine a correct solution

## *Sample Solution for Written-Response Question*

*Use the following information to answer the next question.*

The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—5 marks**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

**A POSSIBLE SOLUTION**

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | **1 259.71** |
| 4 | **1 59.712** | **1 121.14** |

*Use the following additional information to answer the next part of the question.*

When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

- **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

**A POSSIBLE SOLUTION**

Since
$A = 1\ 121.14,\ \ P = 1\ 000,\ \ n = 4$

$$\mathrm{A} = P(1 + i)^n$$

So, $$1\ 121.14 = 1\ 000(1 + i)^4$$

or $$i = \sqrt[4]{\frac{1\,121.14}{1\,000}} - 1$$

$$\doteq 0.028999 \text{ (as a decimal)}$$

As a percentage, rate of return is 2.9%.

**or**

$$\begin{aligned} 1.12114 &= (1 + i)^4 \\ \log(1.12114) &= 4 \log (1 + i) \\ 0.01241\ldots &= \log (1 + i) \\ 1 + i &= 10^{0.01241\ldots} \\ &= 1.028999\ldots \\ i &\doteq 0.028999 \end{aligned}$$

$$\begin{aligned} t_n &= ar^{n-1} \\ 1\ 121.142 &= 1\ 000r^4 \\ 1.12114 &= r^4 \\ 1.028999\ldots &= r \end{aligned}$$

*Use the following additional information to answer the next part of the question.*

A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\,500(0.97)^n,\ n \in N$.

- **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

**A POSSIBLE SOLUTION**

$$\begin{aligned} 1\,250 &= 2\,500(0.97)^n \\ 0.5 &= (0.97)^n \\ \frac{\log(0.5)}{\log(0.97)} &= n \\ n &\doteq 22.756 \end{aligned}$$

It will take 23 years for the investment to decrease to less than $1 250.

- **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include
  - – the function or functions that would be graphed
  - – an appropriate window setting
  - – a summary of how you would use the graph or graphs to determine a correct solution

**A POSSIBLE SOLUTION**

$$\begin{aligned} y_1 &= 2\,500(0.97)^x \\ y_2 &= 900 \end{aligned}$$

Window: $x$: [0, 40, 5] $y$: [0, 2 600, 300]

Calculate the $x$-coordinate of the intersection point and then round the $x$ value up so that the amount is less than $900.

# *Scoring Written-Response Questions*

## Sample 1

*Use the following information to answer the next question.*

The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—10%**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | 1259.71 |
| 4 | 1259.71 | 1121.14 |

*Use the following additional information to answer the next part of the question.*

When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

• **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

Compound Investment Formula: $A = P(1+i)^n$ where A= Amount ($) P= initial amount i= average compounded rate of return (%) n= number of interest terms (a).

let $A = 1121.14$, $P = 1000$, $n = 4$, and solve for $i$:

$1121.14 = 1000(1+i)^4$

$1.12114 = (1+i)^4$

$\sqrt[4]{1.12114} = 1 + i$

$i = \sqrt[4]{1.12114} - 1$

$i \doteq 0.028999$

$\therefore i \doteq 0.029$

$i \doteq 2.9\%$

∴ The average compounded rate of return over the four-year cycle was 2.9% per year.

2

*Use the following additional information to answer the next two parts of the question.*

A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\,500(0.97)^n$, $n \in N$.

• **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

let $R = \frac{1}{2}(2500)$, and solve for $n$:

$\frac{1}{2}(2500) = 2500(0.97)^n$

$1250 = 2500(0.97)^n$

$\frac{1}{2} = (0.97)^n$

$\frac{\log(\frac{1}{2})}{\log(0.97)} = n$

$n = 22.756\,573\,0\ldots$

$n \doteq 23$

∴ The fewest number of years that it would take for the original investment to decrease to less than half of its value is 23 years.

• **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include

– the function or functions that would be graphed

– an appropriate window setting

– a summary of how you would use the graph or graphs to determine a correct solution

Given: $R = 2500(0.97)^n$, solve for n when $R = 900$

let $n = x$ and $R = y$

In "y = " Menu, enter:

$y_1 = 2500(0.97)^x$

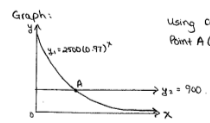
$y_2 = 900$

Window:

x: [0, 150, 50]

y: [0, 2600, 100]

Graph:




Using calc → intersect: Point A (33.541623, 900)

Answer: The fewest number of years it would take for the investment to decrease to less than $900 is 34 years.

Summary: let the function, $2000(0.97)^x$ be $y_1$, and 900 be $y_2$, then using systems of equations, find the intersection of the two graphs where $y_1 = y_2$ to solve for x. Since x is the number of investment years, the solved x value is the answer to the problem. To solve the systems graphically, use the "calc" operation, and select "intersect". This will solve for the point of intersection.

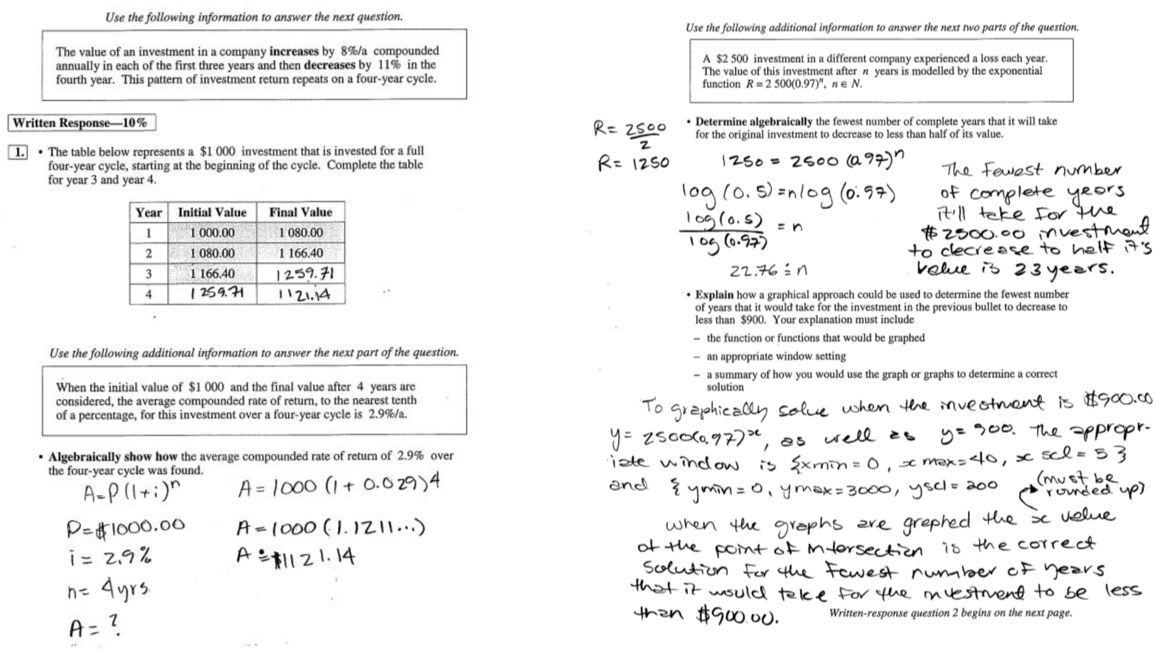
*Written-response question 2 begins on the next page.*

**Score – 5**

**Rationale**

This response has met the *standard of excellence*. All parts of the question have been correctly completed with supporting details, and answers are clearly communicated in a way “that reflects a thorough understanding of the problem.” This student clearly showed all the steps required for an algebraic solution required in bullet 2 and bullet 3. In bullet 4, the student clearly indicated the functions, with the proper window setting, that were needed to solve the question. Any window setting that included the point (33.5, 900) was accepted as a correct response. The summary explaining how the graphs would be used to determine a correct solution had to include or suggest that the *x*-coordinate of the intersection point of the two graphs would give an approximate number of years to reach a value of $900. This *x* value would then have to be rounded up to the nearest whole number of years in order for the investment to decrease to less than $900. To get a full mark for bullet 4, the student was expected to explain more than a series of keystrokes on the calculator.

## Sample 2



*Use the following information to answer the next question.*

The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—10%**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | 1259.71 |
| 4 | 1259.71 | 1121.14 |

*Use the following additional information to answer the next part of the question.*

When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

• **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

$A = P(1+i)^n$
$P = \$1000.00$
$i = 2.9\%$
$n = 4\text{yrs}$
$A = ?$

$A = 1000(1+0.029)^4$
$A = 1000(1.1211\ldots)$
$A \doteq \$1121.14$

*Use the following additional information to answer the next two parts of the question.*

A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\,500(0.97)^n$, $n \in N$.

• **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

$R = \frac{2500}{2}$
$R = 1250$

$1250 = 2500(0.97)^n$
$\log(0.5) = n\log(0.97)$
$\frac{\log(0.5)}{\log(0.97)} = n$
$22.76 \doteq n$

The Fewest number of complete years it'll take for the $2500.00 investment to decrease to half it's value is 23 years.

• **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include

– the function or functions that would be graphed
– an appropriate window setting
– a summary of how you would use the graph or graphs to determine a correct solution

To graphically solve when the investment is $900.00 $y = 2500(0.97)^x$, as well as $y = 900$. The appropriate window is {xmin = 0, xmax = 40, xscl = 5} and {ymin = 0, ymax = 3000, yscl = 200} when the graphs are graphed the x value of the point of intersection is the correct solution (must be rounded up) for the fewest number of years that it would take for the investment to be less than $900.00.

*Written-response question 2 begins on the next page.*

**Score – 4**

**Rationale**

In this response, the student provided a complete solution that reflected a good understanding of the problem. The response is at the *standard of excellence* level based on the student's mathematical processes, organization, and communication strategies; however, the verification of 2.9% rather than an algebraic proof in bullet 2 resulted in a mark of 4 rather than 5.

## Sample 3

*Use the following information to answer the next question.*

The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—10%**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | 1259.71 |
| 4 | 1259.71 | 1121.14 |

1166.40 · 0.08 = 93.312
1166.40 + 93.312 = 1259.712
1259.71 · 0.11 = 138.56832
1259.71 − 138.56832
= 1121.14

*Use the following additional information to answer the next part of the question.*

When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

• **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

$A = A_0 (i)^{t/p}$ (final amount; initial amount; interest; time; period)

$\frac{1121.14}{1000} = \frac{1000 (i)^{4/1}}{1000}$

$1.12 = i^4$

$\log 1.12 = \log i^4$

$\log 1.12 = 4 \log i$

$\frac{\log 1.12}{4} = \frac{4 \log i}{4}$

$0.012... = \log i$

$10^{0.012} = i$

$i = 1.028999866$

interest increasing since there is a positive 1

$1.028999866 - 1 = 0.028999021$

$0.028999021 \times 100 = 2.9\%$



*Use the following additional information to answer the next two parts of the question.*

A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\,500(0.97)^n$, $n \in N$.

• **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

$2500 \div 2 = 1250 - 1 = 1249$

less than half → $\frac{1249}{2500} = \frac{2500 (0.97)^n}{2500}$

$0.4996 = (0.97)^n$

$\log 0.4996 = \log 0.97^n$

$\frac{\log 0.4996}{\log 0.97} = \frac{n \log 0.97}{\log 0.97}$

$n = 22.78$

$n = 22$ years

The fewest number of complete years it will take to decrease the original investment to less than half of its value will be 22 years.

• **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include

- the function or functions that would be graphed
- an appropriate window setting
- a summary of how you would use the graph or graphs to determine a correct solution

First you would plug in the function $R = 2500(0.97)^n$ into your y = function. Since we are looking for the number of years the investment will take to reach less than $900 we must have a window setting that will show all of the data we need. The window settings needed to display the data we require would be x:[0, 50, 5] y:[100, 2500, 500]. To use this graph to find a solution we must also plug in 900 into our y = function so we get a line on the graph where y = 900. Next we find where the two graphs intersect each other and with that value we are able to find that it takes approximately 33 years for the investment to decrease to less than $900.

*Written-response question 2 begins on the next page.*



**Score – 4**

**Rationale**

This student's response met the *standard of excellence*. The solutions were presented in an organized manner; however, a rounding error in bullet 3 and a weak explanation in bullet 4 prevented the student from getting full marks.

## Sample 4

*Use the following information to answer the next question.*

The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—10%**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | 1259.71 |
| 4 | 1259.712 | 1121.14 |

*Use the following additional information to answer the next part of the question.*

When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

• **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

0.08% · 0.08% + 0.08% − 0.11% = ____

*Use the following additional information to answer the next two parts of the question.*

A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\,500(0.97)^n$, $n \in N$.

• **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

$R = 2500(0.97)^n$

$\frac{1250}{2500} = \frac{2500(0.97)^n}{2500}$

$0.5 = 0.97^n$

$\frac{\log 0.5}{\log 0.97} = n \frac{\log 0.97}{\log 0.97}$ $n = 22$ years

It will take 22 years for this investment to decrease to less than half of its value.

• **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include

– the function or functions that would be graphed

– an appropriate window setting

– a summary of how you would use the graph or graphs to determine a correct solution

Functions

$y = 2500(0.97)^x$

$y = 900$

Window Settings

xmin = 0
xMax = 500
xscl = 1
yMin = 0
yMax = 1000
yscl = 1
xres = 1

As the original exponential function of the original function declined eventually it would reach 900 dollars. With that in mind a second function: y = 900 will intersect the original function, I then used the intersect function on my calculator and found the intersection point of the 2 graphs. the x-coordinate of the insection point would be the time in years of when the original investment would be 900 dollars. The correct time in years is 33 years.

*Written-response question 2 begins on the next page.*

**Score – 3**

**Rationale**

This response has met the *acceptable standard*. This student has correctly completed bullet 1 and bullet 4. In bullet 3, the student showed a correct algebraic solution but rounded incorrectly.

## Sample 5

*Use the following information to answer the next question.*

> The value of an investment in a company **increases** by 8%/a compounded annually in each of the first three years and then **decreases** by 11% in the fourth year. This pattern of investment return repeats on a four-year cycle.

**Written Response—10%**

**1.** • The table below represents a $1 000 investment that is invested for a full four-year cycle, starting at the beginning of the cycle. Complete the table for year 3 and year 4.

| Year | Initial Value | Final Value |
|---|---|---|
| 1 | 1 000.00 | 1 080.00 |
| 2 | 1 080.00 | 1 166.40 |
| 3 | 1 166.40 | 1259.71 |
| 4 | 1 259.71 | 1121.15 |

*Use the following additional information to answer the next part of the question.*

> When the initial value of $1 000 and the final value after 4 years are considered, the average compounded rate of return, to the nearest tenth of a percentage, for this investment over a four-year cycle is 2.9%/a.

• **Algebraically show how** the average compounded rate of return of 2.9% over the four-year cycle was found.

A = ?
p = $1000
i = 2.9%/a
n = 4 years

$A = p(1+i)^n$
$A = \$1000(1+0.029)^4$
$A = \$1000(1.029)^4$
$A = (\$1029)^4$
$A = \$10\ 290\ 000$

The average compounded rate of return of 2.9% over the four-year cycle is

*Use the following additional information to answer the next two parts of the question.*

> A $2 500 investment in a different company experienced a loss each year. The value of this investment after $n$ years is modelled by the exponential function $R = 2\,500(0.97)^n$, $n \in N$.

• **Determine algebraically** the fewest number of complete years that it will take for the original investment to decrease to less than half of its value.

$R = 2500(0.97)^n$
$1250 = 2500(0.97)^n$
$y = 2500(0.97)^n - 1250$
($n$ = 23 years)

It would take 23 years.

• **Explain** how a graphical approach could be used to determine the fewest number of years that it would take for the investment in the previous bullet to decrease to less than $900. Your explanation must include

- the function or functions that would be graphed
- an appropriate window setting
- a summary of how you would use the graph or graphs to determine a correct solution

• $R = 2500(0.97)^n$
$900 = 2500(0.97)^n$
$-900 \qquad -900$
$0 = 2500(0.97)^n - 900$
$y = 2500(0.97)^n - 900$ is the function that would be graphed.

• x: [ ]
y: [ ]

• I would put the function into the y= place on my calculator. Then I would graph the function. I would go to 2nd function - calc to determine the x-intercepts or zeros. The x-intercept would end up to turn out to 33.5. The correct solution is 34 years.

*Written-response question 2 begins on the next page.*

**Score – 2**

**Rationale**

In this response, the student “applies some relevant knowledge to find partial solutions; however, the response reflects a basic misunderstanding” of algebra in bullet 2 and bullet 3. The student correctly completed bullet 1, but the omission of a window setting in bullet 4 kept this student from getting full credit for this bullet. The markers felt that this student did not achieve the *acceptable standard* because of omissions and the lack of algebra shown.

## *Suggested Mathematics and Science Directing Words*

| | |
|---|---|
| **Discuss** | The word "discuss" **should not** be used as a directing word on mathematics and science classroom assessments because it is not used consistently to mean a single activity. |
| | ***The following words are specific in meaning.*** |
| **Algebraically** | Using mathematical procedures that involve letters or symbols to represent numbers |
| **Analyze** | To make a mathematical, chemical, or methodical examination of parts to determine the nature, proportion, function, interrelationship, etc., of the whole |
| **Compare** | Examine the character or qualities of two things by providing characteristics of both that point out their mutual *similarities* and *differences* |
| **Conclude** | State a logical end based on reasoning and/or evidence |
| **Contrast/Distinguish** | Point out the *differences* between two things that have similar or comparable natures |
| **Criticize** | Point out the *merits* and *demerits* of an item or issue |
| **Define** | Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits |
| **Describe** | Give a written account or represent the characteristics of something by a figure, model, or picture |
| **Design/Plan** | Construct a plan, i.e, a detailed sequence of actions, for a specific purpose |
| **Determine** | Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and calculations |
| **Enumerate** | Specify one by one or list in concise form and according to some order |
| **Evaluate** | Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages |
| **Explain** | Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail |
| **Graphically** | Using a drawing that is produced electronically or by hand, and that shows a relation between certain sets of numbers |

| | |
|---|---|
| **How** | Show in what manner or way, with what meaning |
| **Hypothesize** | Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically |
| **Identify** | Recognize and select as having the characteristics of something |
| **Illustrate** | Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram |
| **Infer** | Form a generalization from sample data; arrive at a conclusion by reasoning from evidence |
| **Interpret** | Tell the meaning of something; present information in a new form that adds meaning to the original data |
| **Justify/Show How** | Show reasons for or give facts that support a position |
| **Model** | Find a model (in mathematics, a model of a situation is a pattern that is supposed to represent or set a standard for a real situation) that does a good job of representing a situation |
| **Outline** | Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps |
| **Predict** | Tell in advance on the basis of empirical evidence and/or logic |
| **Prove** | Establish the truth or validity of a statement for the general case by giving factual evidence or logical argument |
| **Relate** | Show logical or causal connection between things |
| **Sketch** | Provide a drawing that represents the key features of an object or graph |
| **Solve** | Give a solution for a problem; i.e., explanation in words and/or numbers |
| **Summarize** | Give a brief account of the main points |
| **Trace** | Give a step-by-step description of the development |
| **Verify** | Establish, by substitution for a particular case or by geometric comparison, the truth of a statement |
| **Why** | Show the cause, reason, or purpose |

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