STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME 1. Compare and order integers. [R, V] (7–12)

MANIPULATIVES

- Number lines
- Thermometer

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 88–90
- *Mathpower 7*, pp. 62–63
- *Minds on Math 7*, pp. 262–267
- *Minds on Math 8*, pp. 264–267
- *TLE 7*, Introduction to Integers, Student Refresher pp. 18–19, Teacher's Manual pp. 48–51

Previously Authorized Resources

- *Journeys in Math 8*, pp. 275–277
- *Journeys in Math 9*, pp. 35–37

TECHNOLOGY CONNECTIONS

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Visual and concrete materials are useful for integer instruction.

Demonstrate to students that many real-life situations involve integers. In order to help students understand integers and their operations, use relevant real-world applications, such as temperature, altitude, banking, games and sports.

1. Write the integer that corresponds to the phrase, or write a phrase that corresponds to the integer.

word phrases	integer
a loss of $5	
move 6 units right	
positive 3	
	$^{-}16$
12° below zero	
	+7
18 m above sea level	
3 under par	
	$^{-}19$
	+212

2. Draw a number line from –10 to +10. Circle –8, –5, +2 and +7 on the number line.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

3. Using a number line, write an integer that satisfies:
    - 4 greater than $^{-}3$
    - 2 less than 0
    - between $^{-}3$ and +2
    - 2 units to the left of $^{-}1$
    - 3 units to the right of $^{-}1$

4. Compare the following integers, using “greater than”, “less than” and “>”, “<”.

    Compare:
    - $^{-}4$, 7
    - $^{-}5$, $^{-}17$
    - 3, $^{-}3$
    - 0, $^{-}6$

5. Arrange the following integers from smallest to largest.
    - $^{-}6$, 7, $^{-}1$, 0, 3, 5, $^{-}4$
    - 17, $^{-}21$, 42, $^{-}3$, 92
    - $^{-}29$, 19, $^{-}9$, $^{-}49$, 79

Strand: Number (Number Concepts)
Specific Outcome: 1. Compare and order integers. [R, V] (7–12)

Teaching Notes

Don’t use vector quantities, such as velocity, where the negative sign means direction, or scalar quantities, such as mass, where the negative sign is meaningless.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Performance

1. Show on a number line why $^{-}3$ is less than $^{-}1$.
2. Use a number line to arrange 4, $^{-}6$, 0, $^{-}2$.

Journal Entry

1. Explain why a negative number farther left from 0 is smaller than a negative number closer to 0.
2. Explain why a positive number is always greater than a negative number.

Interview

1. Explain why a golfer likes a negative golf score.
2. Use integers to talk about altitudes.
3. Use integers to talk about bank accounts.

Portfolio

1. Use newspapers or magazines to find articles that include integers; e.g., golf scores, hockey plus/minus, stock market.

Paper and Pencil

1. Plot points +5 and $^{-}5$ on a number line. What do you notice about them? Why do you think number pairs such as $^{-}5$ and +5 are called opposites?
2. Write an integer for each of the following situations:
   a. A person walks up 9 flights of stairs.
   b. An elevator goes down 7 floors.
   c. The temperature falls by 17 degrees.
   d. Sue deposits $150 in the bank.
   e. The peak of the mountain is 2023 m above sea level.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME	Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.
SPECIFIC OUTCOME	2. Find and be able to model an understanding of common multiples, common factors, lowest common multiples, greatest common factors as it applies to whole numbers. [C, PS, R, V] (6–4)

MANIPULATIVES

- Timetable grid

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Mathpower 7*, pp. 176–177
- *Mathpower 9*, pp. 182–186
- *Minds on Math 7*, pp. 92–102
- *Minds on Math 8*, p. 154
- *TLE 7*, Fraction Conversions, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35

Previously Authorized Resources

- *Journeys in Math 8*, pp. 134–135
- *Journeys in Math 9*, pp. 10–11

TECHNOLOGY CONNECTIONS

- Scientific calculator
- Graphing calculator (optional)

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	An understanding of common factors and greatest common factors (GCF) will be useful to students in reducing fractions to lowest terms and in problem-solving situations. It is often useful to pose a problem as a starting point for instruction; e.g.: • Sue's uncle donated 96 juice boxes and 64 chocolate treats for her party. What is the largest number of people that can be at the party and share the food equally (without breaking treats)? Students worked with factors, common factors and prime numbers in previous grades. It may be necessary to review the terms factor, common factor and prime number before extending to prime factorization. Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*. 1. Have students use the "splitting into equal groups" model of division to work out the answers to the following questions. • 8 ÷ 2 = ? • 6 ÷ 2 = ? • 4 ÷ 2 = ? • 2 ÷ 2 = ? • 0 ÷ 2 = ?

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Have them discuss the last situation: 0 objects to split up with 2 in each group.

Then have them consider 2 ÷ 0 (2 objects split up with 0 in each group). Most students will see the question itself as being silly and, therefore, there is no answer.

- *Activity*: Building patterns on a number chart

  Goal: To explore number concepts and their relationships, such as:

  - primes and composites
  - multiples and factors
  - odd and even numbers
  - divisibility rules for 2, 3, 4, 5, 9

  Materials: Each student receives

  - a number chart 1 to 100
  - colour pencils or markers (7 different colours)

  Student instructions:

  1. Place a **yellow** check mark on all multiples of 2. Look at the pattern and describe it. (*Possible student descriptions*: a) Alternate squares are filled. b) It looks like a checkerboard. c) Those are even numbers. They are multiples of 2.)
  2. Continue by placing a **red** check mark on all multiples of 3. Look at the pattern and describe it. (*Possible student descriptions*: a) They are even and odd numbers. b) Some numbers have both red and yellow checks. c) Some numbers have two factors.)
  3. Continue by placing a **blue** check on all multiples of 4. Look at the pattern and describe it. (*Possible student descriptions*: a) All are even. b) Some numbers have all 3 colours. c) Some numbers have 3 factors.)
  4. Continue filling in the pattern in the following way:
     purple — multiples of 5
     green — multiples of 6
     orange — multiples of 7
     black — multiples of 8

  Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

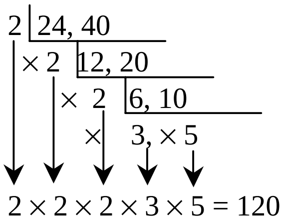
2. Lowest Common Multiple (LCM)

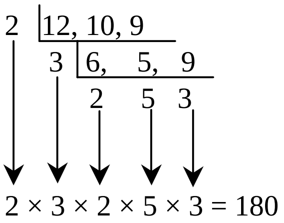
2.1

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Cut the grid into horizontal strips. To find the LCM for 3 and 5, place the 3 and 5 strips together and check the lowest number common to both strips.

2.2 To find the LCM, use repeated division by prime factors.





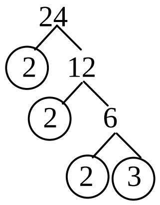
2.3 Use a calculator to generate multiples of a number by keying in the number and the + key repeatedly. **Note**: Different procedures apply to different calculators.

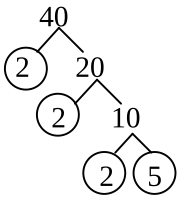
Extension: Using the TI-83 calculator, select
Math → Number
↓
LCM (
↓
enter the 2 numbers separated by a comma. Close bracket. ENTER.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2.4 Use a prime factorization.





24 = 2 × 2 × 2 × 3
40 = 2 × 2 × 2 × 5

↓ ↓ ↓

LCM = 2 × 2 × 2 × 3 × 5
= 120

3. Greatest Common Factor (GCF)

3.1 To find the GCF of 24 and 36 using the listing of factors method, list the factors for each number.

24 = 1, 2, 3, 4, 6, 8, 12, 24
36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Circle the factors in common and select the greatest common factor, which in this case is 12.

3.2 To find the GCF, use repeated division by prime factors.

3 | 36, 24
2 | 12, 8
2 | 6, 4
1 | 3, 2

GCF = 3 × 2 × 2 × 1 = 12

3.3 The prime factorization method should also be considered in determining the GCF. To find the GCF of 24 and 36 using prime factorization, first write the prime factors for each number:
24 = 2 × 2 × 2 × 3
36 = 2 × 2 × 3 × 3

Then choose factors common to both numbers and multiply them to get the GCF. The GCF of 24 and 36 is 2 × 2 × 3 = 12.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Extension: Using the TI-83 calculator, select
Math → Number
↓
GCF (
↓
enter the 2 numbers separated by a comma. Close bracket.
ENTER.

Application Questions

1. Three sisters, Janice, Jackie, and Gaye, live in 3 different cities. Janice returns home every 2 years, Jackie returns home every 3 years, and Gaye every 5 years. If the sisters were last reunited in 1990, what year will they get together next?

   Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

Suggestion: Read *Spaghetti and Meatballs for All!: A Mathematical Story* by Marilyn Burns.

2. You are making reservations for a dinner party at a local restaurant. The restaurant can sit the guests at tables of 2, 3, 4 or 5. If all the places at a table must be filled and the maximum number of guests the restaurant can accommodate is 28, what is the maximum number of guests you can invite to the dinner party for each of the above seating arrangements?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes Questions should include components that do and do not require calculators.	see below

Paper and Pencil ❶

1. Chris is filling loot bags for his little sister's birthday party. He has 24 toys, 36 caramels and 60 chocolate treats. What is the largest number of bags that can be filled if all the treats are to be used, no treats are subdivided, and all children receive the same items in their loot bags?

2. John is creating a miniature, quilted wall hanging made of square blocks. He wants the wall hanging to be exactly 15 cm by 20 cm.
   a. Find the size of the largest finished square block with whole number side lengths that can be used to exactly cover the area.
   b. Find the GCF of 15 and 20.
   c. Compare your answers in part a and part b, and describe what you notice.

3. Sarah decides to make a quilt for her bed based on the design John used in question 2. However, she feels that she should enlarge the block size so that it will not require as many blocks. The bed quilt must be 200 cm by 250 cm.
   a. Make a list of possible sizes for the square blocks.
   b. What is the largest square block that can be used?
   c. Sarah decided that the blocks should be larger than 15 cm but smaller than 30 cm. What size would the finished blocks need to be to make this work?

4. The GCF of 8 and an unknown number is 4. Find three possible values for the missing number. Describe what all the values have in common.

5. Bill and Jenny regularly exercise at a local gymnasium. Bill exercises one day out of every 6, and Jenny exercises one day out of every 4. If they both start today, and the gymnasium is open every day, how many days will they exercise on the same day in a 5 week span?

6. Find the LCM for:
   a. 9, 12, 15
   b. 5, 10, 20

7. Find the GCF for:
   a. 36, 42, 60
   b. 40, 50, 65

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❶ Paper and Pencil questions 1 to 4 and 8 to 11 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

8. a. The LCM of two numbers is 24. Find all possible values for the numbers.
   b. The GCF of 8 and an unknown number is 4, and the LCM of the two numbers is 24. Find a possible value for the unknown number.

9. If the GCF of two numbers is 8, and the LCM is 80, what are the possibilities for the pair of numbers?

10. Joe and Pat work part-time at a music rental store. Joe works once every four days, and Pat works once every six days.
    a. If they both started work on September 27, and the store is open every day, what will be the next date they work together?
    b. Find two more dates on which they will work together.

11. Solve the following, using a number line.

    Sarah has three aunts who live in other provinces of Canada. Her aunt who lives in Vancouver visits every fourth summer, her aunt who lives in Calgary visits every third summer, and her aunt who lives in Toronto visits every second summer. The family had a reunion when Sarah was 6 years old. Sarah's dad is planning another reunion when all his sisters visit again. How old will Sarah be at the next reunion?

Interview

1. Ask students why the GCF has to be a factor of the LCM.

Journal Entry

1. Explain how you would find the LCM for 4, 7 and 5.

Performance

1. Have students find LCMs using multiple strips.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME 3. Demonstrate and explain the meaning of proper and improper fractions. [C, R, V] (6–9)

MANIPULATIVES

- Fraction blocks
- Fraction strips
- Fraction circles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, p. 63
- *Mathpower 7*, pp. 48–49
- *Minds on Math 7*, pp. 92–102
- *TLE 7*, Fraction Conversions, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35 (review only)

Previously Authorized Resources

- *Journeys in Math 8*, pp. 142–144
- *Journeys in Math 9*, pp. 20–21

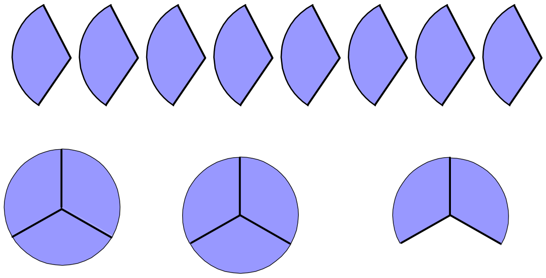
TECHNOLOGY CONNECTIONS

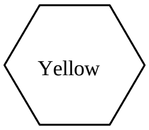
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes It is important that students understand the concepts associated with fractions before they develop calculation skills using algorithms. Otherwise students don't understand why the algorithms apply. When students participate in activities using symbols, they should be able to use manipulatives to confirm their answers. Visualization is important in developing the meaning of fractions. Building patterns from blocks can be very effective.	This outcome should not require a great deal of time. Use common everyday situations, such as the following, as the focus for informal class discussion to determine what students already know about fractions. You and a friend ordered two pizzas, each cut into 6 equal slices. You eat all of your pizza and one slice from your friend's pizza. Express the amount of pizza you ate as an improper fraction. If your friend ate the remaining pizza, express the amount she ate as a proper fraction.

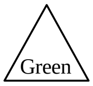
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

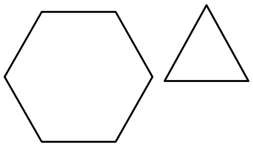
Teaching Notes

1. Use fraction circles to a show that $\frac{8}{3}$ is equivalent to $2\frac{2}{3}$.

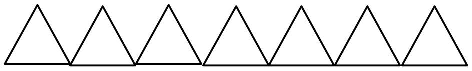


2. Using pattern blocks, let  represent 1 whole.

Then, if  represents $\frac{1}{6}$, illustrate $1\frac{1}{6}$ as a mixed number and as an improper fraction.



$1\frac{1}{6}$



$\frac{7}{6}$

Strand: Number (Number Concepts)
Specific Outcome: 3. Demonstrate and explain the meaning of proper and improper fractions. [C, R, V] (6–9)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

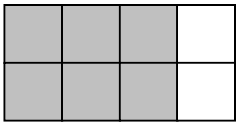
Teaching Notes

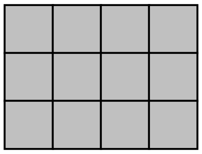
In order to relate improper fractions to a number line, the relationship between improper fractions and mixed numbers should be developed.

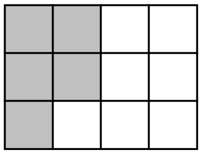
Paper and Pencil

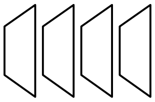
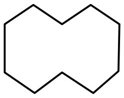
1. The complete rectangle represents one whole. For each diagram, what fraction is represented by the shaded portions?

a. 

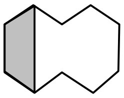
b. 

c. 

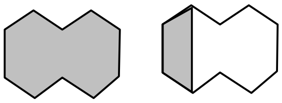
d. 

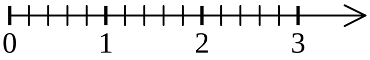
2. If  are needed to build , then:

a. What fraction of the whole does the shaded portion represent?



b. What fraction of the whole does the shaded portion below represent? (answer in improper fraction form)

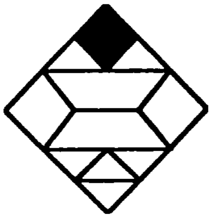


3. 

Show the position that represents:

a. $\frac{3}{5}$

b. $\frac{12}{5}$



4. The sketch to the right shows the pattern for a stained glass project.

a. If the shaded portion of the pattern uses $\frac{1}{3}$ of a pane of coloured glass, how many panes of glass will be used in completing this design?

Performance

1. Use pattern blocks, where the value of the yellow hexagon is one whole, to make a pattern with a value of $2\frac{2}{3}$.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME

4. Distinguish between exact values and decimal approximations of square roots and cube roots. [E, T] (8–8)

MANIPULATIVES

- Square tiles
- Grid paper

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 18–20
- *Interactions 8*, pp. 42–52
- *Interactions 9*, pp. 43–44, 84–85
- *Mathpower 8*, pp. 28–31
- *Mathpower 9*, pp. 8–11
- *Minds on Math 9*, pp. 300–304
- *TLE 8*, Square Roots, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35
- *TLE 9*, Square Roots, Student Refresher pp. 6–7, Teacher's Manual pp. 24–27
- *TLE 10*, Approximating Irrational Numbers, Student Refresher pp. 8–9, Teacher's Manual pp. 28–31 (the only place for cube roots)

Previously Authorized Resources

- *Journeys in Math 8*, pp. 18–19
- *Journeys in Math 9*, pp. 106–110

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Students should be aware of the perfect squares from 1 to 625. They should know the common squares and at least recognize that a number is a perfect "square." For example, students should know $25^2 = 625$ and should recognize 289 as being a perfect square. Some time should be spent on this before beginning the outcome. This is used again in working with the Pythagorean theorem.	A simple introduction to this activity is to have students compare their calculator answers to the square roots of perfect squares and of non-perfect squares. Some will round at a different number of digits and show that these are approximations. Another option is to have students write down the calculator display for $\sqrt{2}$ (1.414 …), clear the calculator, and then use the calculator to multiply that number by itself. This sometimes can demonstrate that the decimal value is not exact.

Strand: Number (Number Concepts)
Specific Outcome: 4. Distinguish between exact values and decimal approximations of square roots and cube roots. [E, T] (8–8)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

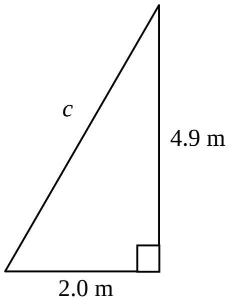
Teaching Notes

1. A player at third base must throw the ball to home plate. If the area of a square baseball diamond is 751 m^2, what is the distance from third base to home plate?

   Since
   $$\begin{aligned} A &= S^2 \\ 751 &= S^2 \\ \sqrt{751} &= S \\ 27.4 &\doteq S \end{aligned}$$

   The approximate distance is 27.4 m, which is $\sqrt{751}$ m rounded to the nearest tenth. The exact distance is $\sqrt{751}$ m.

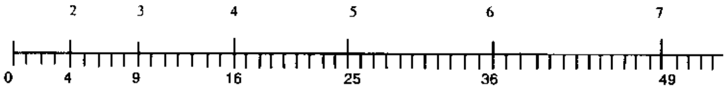
2. A power pole that is 4.9 m high is held up by a guy wire anchored 2.0 m from its base. How long is the guy wire?



   $$\begin{aligned} a^2 + b^2 &= c^2 \\ 4.9^2 + 2.0^2 &= c^2 \\ 28.01 &= c^2 \\ \sqrt{28.01} &= c \\ 5.3 &\doteq c \end{aligned}$$

   The guy wire is approximately 5.3 m long. The exact measure is $\sqrt{28.01}$ m.

3. Have students use known squares to estimate square roots; e.g.: Make a number line such as the following:



   To estimate the square root of 28, find it on the number line and decide where it is in relation to the nearest squares and corresponding square roots. For example, $\sqrt{28}$ is less than 5.5 and is approximately equal to 5.3.

4. Have students use known cubes to estimate cube roots.

Number	1	2	3	4
Cube	1	8	27	64

   The cube root of 35 is between 3 and 4 since 35 lies between 27 and 64.

Strand: Number (Number Concepts)
Specific Outcome: 4. Distinguish between exact values and decimal approximations of square roots and cube roots. [E, T] (8–8)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

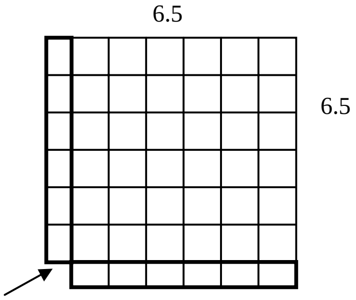
Teaching Notes

The following activity may be used to approximate the square root of a number. However, it requires a considerable amount of time.

1. Using grid paper (1 cm × 1 cm), mark out an area that is 6 squares by 6 squares. Cut an additional strip that is 1 cm × 6 cm. How many squares do you have in all? (42)

   Cut the 1 cm × 6 cm strip in half lengthwise, and align these two strips on adjacent sides of your original square. Use the result to estimate $\sqrt{42}$ (Diagram 1).

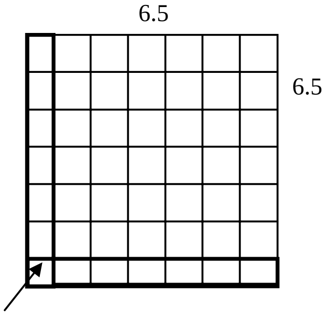
Diagram 1



This does not quite make a complete square, so $\sqrt{42} \cong 6.5$ but is slightly less than 6.5.

Use a similar procedure to estimate $\sqrt{43}$ (Diagram 2).

Diagram 2



This square is covered twice. Therefore, $\sqrt{43}$ is slightly more than 6.5.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME 5. Differentiate between principal square root and positive and negative square roots of a number. Give examples where answers would involve the positive (principal) square root, or both positive and negative square roots of a number. [C, CN, PS, R] (9–3)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 18–20, 25
- *Interactions 9*, pp. 84–85
- *Mathpower 8*, pp. 28–31
- *Mathpower 9*, pp. 8–11
- *Minds on Math 9*, p. 306
- *TLE 8*, Square Roots, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35

Previously Authorized Resources
- *Journeys in Math 8*, p. 18
- *Journeys in Math 9*, pp. 106–107
- *Math Matters: Book 2*, pp. 50–53

TECHNOLOGY CONNECTIONS
- Scientific calculator

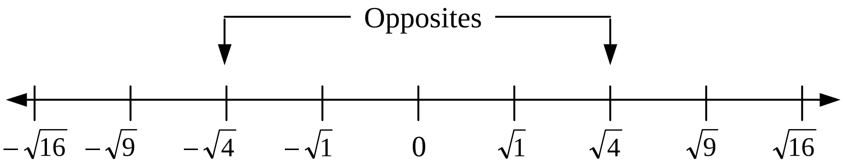
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Recognize that $-\sqrt{4}$ is different from $\sqrt{-4}$. $-\sqrt{4}$ is $^-2$, but $\sqrt{-4}$ is not a real number.	Just as subtraction is the opposite of addition and division is the opposite of multiplication, the opposite of squaring a number is to find the square root. For example, $2^2 = 2 \times 2 = 4$ and $(^-2)^2 = (^-2) \times (^-2) = 4$. Therefore, the square roots of 4 are 2 and $^-2$. Positive numbers always have two square roots: one positive and the other negative. The symbol $\sqrt{4}$ stands for the **positive** square root of 4. The positive square root is also called the **principal** square root. Thus $\sqrt{4} = 2$ The symbol $-\sqrt{4}$ stands for the **negative** square root of 4. $-\sqrt{4} = {}^-2$

Strand: Number (Number Concepts)
Specific Outcome: 5. Differentiate between principal square root and positive and negative square roots of a number. Give examples where answers would involve the positive (principal) square root, or both positive and negative square roots of a number. [C, CN, PS, R] (9–3)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

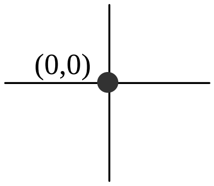
Teaching Notes

From the following number line, $\sqrt{4}$ and $-\sqrt{4}$ are opposites.



1. If you want to find the length of one side of a square garden whose area is 25 m^2, explain why you would use only the positive square root of 25.

2. A square has one vertex at (0, 0) and an area of 25 square units. Find the coordinates of the other vertices for four such squares.



The extension requires Pythagorean theorem.

Extension: There are more squares than the four obvious ones. Find a few of them, and explain how they are found.

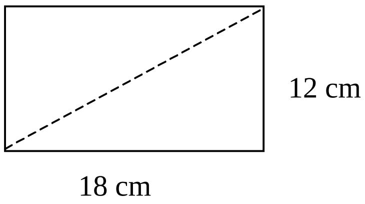
Strand: Number (Number Concepts)
Specific Outcome: 5. Differentiate between principal square root and positive and negative square roots of a number. Give examples where answers would involve the positive (principal) square root, or both positive and negative square roots of a number. [C, CN, PS, R] (9–3)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

1. In each of the following situations, determine if both positive and negative square roots are appropriate or if just the principal square root is appropriate.

   a. Tracy was asked to find the length of the diagonal of this rectangle. Should she use both square roots, or only the principal square root?



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 12^2 + 18^2 &= c^2 \\ 144 + 324 &= c^2 \\ 468 &= c^2 \\ \sqrt{468} &= c \end{aligned}$$

   b. Travis was asked to solve the following equation. His work is shown. Should he use both square roots, or only the principal square root?

$$\begin{aligned} 2x^2 + 17 &= 67 \\ 2x^2 &= 50 \\ x^2 &= 25 \\ x &= \sqrt{25} \end{aligned}$$

   c. The result when 7 is taken away from the square of a number is 18. What are the possible values for the number?

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME Develop a number sense of powers with integral exponents and rational bases.

SPECIFIC OUTCOME 6. Recognize and illustrate the meaning of a power, base, coefficient and exponent, including rational numbers or variables as bases or coefficients. [R, V] (9–4)

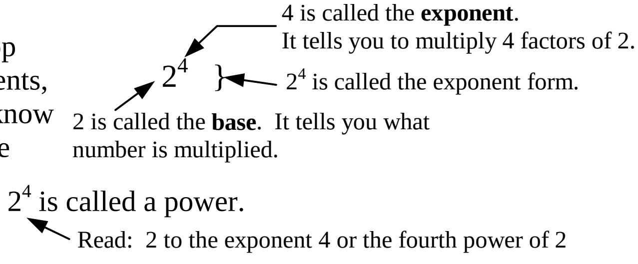
MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 52–55
- *Interactions 7*, pp. 38–39
- *Interactions 8*, pp. 35–40
- *Mathpower 7*, pp. 18–19
- *Mathpower 8*, pp. 4–5
- *Mathpower 9*, pp. 16–18, 26–29
- *Minds on Math 7*, pp. 30–31
- *Minds on Math 8*, pp. 456–461
- *Minds on Math 9*, pp. 276–279, 292–295
- *TLE 7*, Powers and Exponents, Student Refresher pp. 2–3, Teacher's Manual pp. 16–19
- *TLE 9*, Powers, Bases and Exponents, Student Refresher pp. 8–9, Teacher's Manual pp. 28–31
- *TLE 9*, Laws of Exponents 1–2, Student Refresher pp. 10–13, Teacher's Manual pp. 32–39

Previously Authorized Resources
- *Journeys in Math 8*, pp. 16–17
- *Journeys in Math 9*, pp. 94–95

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	**Exponents and Powers** In order to develop skills with exponents, students have to know the names of these parts. 4 is called the **exponent**. It tells you to multiply 4 factors of 2. 2^4 2^4 is called the exponent form. 2 is called the **base**. It tells you what number is multiplied. 2^4 is called a power. Read: 2 to the exponent 4 or the fourth power of 2 $2^4 = 2 \times 2 \times 2 \times 2$ (expanded form) Some powers have common names. $5^2 \leftarrow$ 5 squared $4^3 \leftarrow$ 4 cubed

Strand: Number (Number Concepts)
Specific Outcome: 6. Recognize and illustrate the meaning of a power, base, coefficient and exponent, including rational numbers or variables as bases or coefficients. [R, V] (9–4)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

The numerical factor of a term is called the coefficient. In the expression $2x^4$, the 2 is considered to be the coefficient of the power x^4. The coefficient of the term $-x^2$ is –1.

Term	Coefficient
$2x^4$	2
$3m^2n$	3
$-5y$	–5
$-x$	–1
x	1
$\frac{x^2}{3}$	$\frac{1}{3}$

Examples

1. What is the value of the coefficient in each of the following expressions?
   a. $-x^4$ b. $\frac{x^2}{5}$

Simplify Rational Numbers with Whole Numbers as Exponents

Students need to evaluate powers using pencil and paper.

Examples

1. Evaluate.
   a. 3^2 b. $^-3^2$ c. $(^-3)^2$ d. $\left(\frac{2}{3}\right)^3$ e. $(0.03)^2$ f. $\frac{2^3}{3}$

2. Which is larger, 3^4 or 4^3?

Some older scientific calculators cannot find the power if the base is negative. Most newer calculators, and all graphing calculators, can handle negative bases.

3. Students need to be able to evaluate powers with calculators. Evaluate:
   a. 2.7^8
   b. 389^3
   c. 2^{31}
   d. $^-3^4$
   e. $(^-3)^2$

4. Which of the following symbols, <, > or = would you place between $^-5^2$ and $(^-5)^2$ to make a true statement?

Strand: Number (Number Concepts)
Specific Outcome: 6. Recognize and illustrate the meaning of a power, base, coefficient and exponent, including rational numbers or variables as bases or coefficients. [R, V] (9–4)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

5. Evaluate these powers mentally.
   a. $\left(\frac{2}{5}\right)^2$
   b. $^{-}2^4$
   c. $(0.06)^2$
   d. What is the last digit of 5^8?

Evaluate Algebraic Expressions with Whole Number Exponents

Remind students to replace each variable with a set of parentheses **and then** place the number inside the parentheses.

If $x = {}^{-}3$, evaluate $^{-}x^2$.

$$\begin{aligned} {}^{-}x^2 &= -({}^{-}3)^2 \\ &= -(9) \\ &= {}^{-}9 \end{aligned}$$

Examples

1. If $x = {}^{-}2$, find the value of x^2.
   Answer
   $$\begin{aligned} x^2 &= ({}^{-}2)^2 \\ &= 4 \end{aligned}$$

2. If $x = {}^{-}5$ and $y = {}^{-}3$, find the value of $^{-}2xy^2$.
   Answer
   $$\begin{aligned} {}^{-}2xy^2 &= {}^{-}2({}^{-}5)({}^{-}3)^2 \\ &= {}^{-}2({}^{-}5)(9) \\ &= 90 \end{aligned}$$

3. If $x = 2$ and $y = 3$, find the value of $x^y - y^x$.
   Answer
   $$\begin{aligned} x^y - y^x &= (2)^3 - (3)^2 \\ &= 8 - 9 \\ &= {}^{-}1 \end{aligned}$$

4. If x represents an integer, which integers satisfy:
   a. $x^2 = 1$
   b. $x^2 < 1$
   c. $x^2 > 1$

5. Arrange x^2, x^3 and $^{-}x^3$ in decreasing order, if $x = -\frac{1}{2}$.

Strand: Number (Number Concepts)
Specific Outcome: 6. Recognize and illustrate the meaning of a power, base, coefficient and exponent, including rational numbers or variables as bases or coefficients. [R, V] (9–4)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Find the value of $(x + y)(x^2 – xy + y^2)$, if $x = {}^{-}1$ and $y = 2$.

2. Express the number 48 in exponent form.

3. Which is larger, 5^3 or 3^5?

Performance

1. John wants to use his calculator to find 9^4, but the 4 key is missing.
   a. Explain how he can use the calculator to find the answer to this question, even though the 4 is missing.
   b. Suppose the 9 key is missing instead. Explain how he might now use the calculator to find the answer.

Interview

1. Explain to students that Susan was asked to find the volume of a cube that was 4 cm on one side. She wrote: 4^3 as $3 \times 3 \times 3 \times 3 = 81$. Ask students if the solution is correct and also to explain why or why not.

Portfolio

1. Ask students to find a value for $\square$ and a value for * which would make the following sentence true: $3^{\square} = 9^{*}$. Ask if there are other values for $\square$ and * that would work.

2. Use patterning to help you find the **last** digit in:
   a. 4^{100}
   b. $({}^{-}2)^{101}$
   c. 5^{50}
   Why can't you use your calculator to answer this?

Journal

1. Explain the difference between 2^3 and 3^2.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Develop a number sense of powers with integral exponents and rational bases.

SPECIFIC OUTCOME

7. Explain and apply the exponent laws for powers with integral exponents.

$$x^m \bullet x^n = x^{m+n}$$

$$x^m \div x^n = x^{m-n}$$

$$\left(x^m\right)^n = x^{mn}$$

$$(xy)^m = x^m y^m$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n},\ y \neq 0$$

$$x^0 = 1,\ x \neq 0$$

$$x^{-n} = \frac{1}{x^n},\ x \neq 0$$

[PS, R] (9–5)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 52–58
- *Interactions 9*, pp. 28–42
- *Mathpower 9*, pp. 20–21, 24–43
- *Minds on Math 9*, pp. 276–299
- *TLE 9*, Laws of Exponents 1–3, Student Refresher pp. 10–15, Teacher's Manual pp. 32–43
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–17, Teacher's Manual pp. 44–47

Previously Authorized Resources

- *Journeys in Math 9*, pp. 96–101
- *Math Matters: Book 2*, pp. 95–100

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The exponent laws, using variable bases, are only touched on briefly in the junior high school curriculum. The concentration is on integral bases. As with numerical bases, it is important to develop the laws by using the expansion of powers, then simplifying to illustrate each law; e.g.:

INSTRUCTIONAL STRATEGIES/SUGGESTIONS❶

Teaching Notes

a. $x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) = x^5$

b. $\dfrac{x^5}{x^3} = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^2$

c. $(x^3)^2 = (x^3)(x^3) = (x \cdot x \cdot x)(x \cdot x \cdot x) = x^6$

d. $(xy)^3 = (x \cdot y)(x \cdot y)(x \cdot y) = (x \cdot x \cdot x)(y \cdot y \cdot y) = x^3 y^3$

e. $\left(\dfrac{x}{y}\right)^3 = \left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right) = \dfrac{x^3}{y^3}$

f. $$\frac{x^3}{x^3} \begin{cases} \nearrow\ = x^{3-3} = x^0 \\[2ex] \searrow\ = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = 1 \end{cases} \Bigg\} \text{ therefore, } x^0 = 1$$

g. $$\frac{x^3}{x^5} \begin{cases} \nearrow\ = x^{3-5} = x^{-2} \\[2ex] \searrow\ = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \dfrac{1}{x^2} \end{cases} \Bigg\} \text{ therefore, } x^{-2} = \frac{1}{x^2}$$

1. Write equivalent expressions with positive exponents.

   a. $\dfrac{x^{-2}}{y}$

   b. $\dfrac{x^2}{y^{-3}}$

   c. $\dfrac{x^{-2}y}{z^{-6}}$

   d. $\dfrac{-2x^{-2}}{y^{-4}}$

2. Express $2^{-1} + 4^{-1}$ as the sum of two rational numbers.

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❶ Instructional Strategies/Suggestions questions 1 to 3 are reproduced, by permission, from Manitoba Education and Training. *Senior 1 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Using a variety of operations, write expressions that are equivalent to x^2. Use your imagination. 4. Evaluate: a. 3^{-2} b. -3^{-2} c. $(-3)^{-2}$ d. $\frac{2^{-2}}{3}$ e. $\frac{1}{2^{-3}}$ 5. Simplify, removing negative exponents and parentheses. a. $\left(\frac{2}{3}\right)^{-5}$ b. $\left(35^2\right)^3$ c. $(xy)^m$ d. $\left(\frac{x}{y}\right)^n$ e. $\left(\frac{x}{y}\right)^{-n}$ 6. The quantities in bold type may contain errors. By correctly applying the exponent laws, replace incorrect answers with correct answers. a. $m^4 \bullet m^3 = \boldsymbol{m^{12}}$ b. $m^{12} \div m^2 = \boldsymbol{m^6}$ c. $\left(m^5\right)^2 = \boldsymbol{m^7}$ d. $(xy)^6 = \boldsymbol{x^1}\ y^6$ e. $\left(\frac{x}{y}\right)^3 = \left(\frac{\boldsymbol{x^3}}{\boldsymbol{y^1}}\right)$ f. $\left(\frac{2^3}{3}\right) = \left(\frac{\mathbf{2^3}}{\mathbf{3^3}}\right)$ g. $3^0 = \mathbf{0}$ h. $5^1 = \mathbf{1}$ 7. Explain why $5^0 = 1$, not zero.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Predict and verify to find values for *x*.

   a. $x^4 \times x^2 = 64$
   b. $x^8 \div x^3 = 243$
   c. $(x^2)^4 = 256$
   d. $(x^3)^{-2} = 64$

2. Provide the missing exponents.

   a. $10^{-5} \div 10^5 = 10^{\square}$
   b. $10^{-6} \div 10^{\square} = 10^{-14}$
   c. $\dfrac{10^{12}}{10^{-5}} = 10^{\square}$
   d. $(-5)^{15} = [(-5)^{\square}]^3$
   e. $3^5 \times 4^3 = (3 \times 4)^{\square} = 12^{\square}$
   f. $5 \div (-6)^{-7} = 5^{\square} \times (-6)^{\square}$

Journal

1. Write in your own words what is meant by a power of a power.

2. Copy and complete the following power chart. Describe any patterns you see. Make a similar chart using 3 as the base; compare the charts, and describe their similarities.

   $2^5 = 32$
   $2^4 = 16$
   $2^3 =$
   $2^2 =$
   $2^1 =$
   $2^0 =$
   $2^{-1} = \frac{1}{2}$
   $2^{-2} = \frac{1}{4}$
   $2^{-3} = \frac{1}{8}$
   $2^{-4} =$
   $2^{-5} =$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Interview❶** 1. Ask students to explain two ways that $(2 \times 5)^4$ can be calculated. 2. $2^4 \times 2^{-2} \times 5^3 \times 5^{-3} \times 10^5 \times 10^{-4}$ a. Ask students to explain why this is easy to solve mentally. b. Ask students to solve the problem mentally. c. Ask students to write a similar problem involving six powers that is also easy to solve mentally. Have the students exchange their problems with fellow students.

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❶ Interview questions 1 and 2 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME Use a scientific calculator to solve problems involving real numbers.

SPECIFIC OUTCOME

8. Document and explain the calculator keying sequences used to perform:
    - square roots, cube roots
    - exponents
    - scientific notation
    - sine, cosine, tangent
    - integers.

    [PS, R, T] (9–10)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–19, Teacher's Manual pp. 44–51
- *TLE 9*, Ratios in Right Triangles, Student Refresher pp. 58–59, Teacher's Manual pp. 128–131
- *TLE 9*, Finding Unknown Angles, Student Refresher pp. 62–63, Teacher's Manual pp. 136–139

Previously Authorized Resources

- *Math Matters: Book 2*, pp. 31–53
- *Mathematics 9*, pp. 7, 24, 46, 57, 146, 162, 163, 258, 342

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	This specific outcome can be approached in the appropriate context throughout the course. Students should be able to communicate keying instructions to perform these operations. Since different calculators may require different keying sequences, you may choose to develop the most common sequence(s) as a group. Individual calculator differences may be dealt with separately. Within the context of integer work, the order of operations usually requires appropriate use of parentheses.

Strand: Number (Number Operations)
Specific Outcome: 8. Document and explain the calculator keying sequences used to perform: square roots, cube roots; exponents; scientific notation; sine, cosine, tangent; integers. [PS, R, T] (9–10)

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Example 1. To find the square root of 2, you may input one of these possibilities: a. $\boxed{\sqrt{\ }}$ 2 b. 2 $\boxed{\sqrt{\ }}$ c. $\boxed{2^{nd}}$ $\sqrt{\ }\boxed{x^2}$ 2 d. 2 $\boxed{2^{nd}}$ $\sqrt{\ }\boxed{x^2}$ Each student should be expected to communicate the keystrokes for the calculator he/she uses. In this example, a key is labelled $\boxed{2^{nd}}$. Alternative labels include $\boxed{\text{shift}}$ or $\boxed{\text{INV}}$. Also, students should be reminded to put their calculators in DEGREE mode before finding sine, cosine or tangent.

Strand: Number (Number Operations)
Specific Outcome: 8. Document and explain the calculator keying sequences used to perform: square roots, cube roots; exponents; scientific notation; sine, cosine, tangent; integers. [PS, R, T] (9–10)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Journal

1. Students may keep a section of their notebook to record keystroke sequences for each operation as that operation is developed throughout the course. The keystroke sequence would depend on the calculator that the student uses and the solution strategy used.

Paper and Pencil (calculator)

1. a. Multiply 1 600 000 × 4 000 000.
   How does your calculator show the answer?
   If your calculator displays $\boxed{6.4 \qquad 12}$, what is your answer in scientific notation?

   b. Multiply 0.000 053 × 0.000 000 3.
   If your calculator displays $\boxed{1.59 \qquad -11}$, what is your answer in scientific notation?

2. a. Estimate $(1.9 \times 3.1)^3$

   b. Calculate $(1.9 \times 3.1)^3$, using the keying sequence

   $\boxed{(}$ 1.9 $\boxed{\times}$ 3.1 $\boxed{)}$ $\boxed{y^x}$ 3 $\boxed{=}$

   c. If you calculate without the brackets, do you get the same answer? Explain.

   d. Calculate $(-2)^6$ and -2^6. Compare your answers and explain.

   e. Mentally calculate $\frac{4^2}{2+6}$. Write a keying sequence you could use if you used your calculator.

These keying sequences are not unique, even for the same calculator.

Interview

1. Two students or a student and the teacher exchange calculators. One explains the keying sequence for solving a problem to the other. Then their roles are reversed.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

9. Perform arithmetic operations with integers concretely, pictorially and symbolically. [PS, V] (7–16)

MANIPULATIVES

- Algebra tiles
- Two-sided counters
- Bingo chips
- Number lines
- Thermometers
- Coloured squares

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 94–102, 276–287
- *Mathpower 7*, pp. 116–135
- *Minds on Math 7*, pp. 270–304
- *Minds on Math 8*, pp. 270–275
- *Minds on Math 9*, pp. 26–31
- *TLE 7*, Exploring Integers, Student Refresher pp. 28–29, Teacher's Manual pp. 68–71
- *TLE 7*, Adding Integers, Student Refresher pp. 30–31, Teacher's Manual pp. 72–75
- *TLE 7*, Subtracting Integers, Student Refresher pp. 32–33, Teacher's Manual pp. 76–79

Previously Authorized Resources

- *Journeys in Math 8*, pp. 280–291
- *Journeys in Math 9*, pp. 38–51

TECHNOLOGY CONNECTIONS

- Scientific calculator
- Spreadsheet

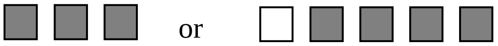
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Manipulatives and concrete models should be used to develop the concept of calculations with integers. Useful manipulatives include algebra tiles, bingo chips, two-sided counters, number lines, thermometers and coloured squares. Results should then be connected to symbolic representations. When using algebra tiles (two coloured items) the zero principle is important; i.e., different colours cancel each other.

Strand: Number (Number Operations)
Specific Outcome: 9. Perform arithmetic operations with integers concretely, pictorially and symbolically. [PS, V] (7–16)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Using algebra tiles, express the number 3 in as many ways as possible; e.g.:



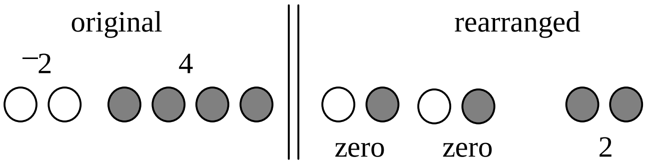
Ask students to represent:

a. ¯4, using 6 coloured counters
b. zero, using 6 coloured counters
c. +2, using 6 coloured counters

Adding Integers

Using Tiles: When adding two integers, it is necessary to first model each integer, then match positive and negative values to make zeros;

e.g., ¯2 + 4



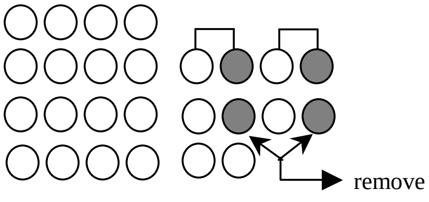
Therefore, ¯2 + 4 = 2 (symbolic connection).

Subtracting Integers❶

If you don't want to use the regrouping method, you will need to use double-sided tiles.

Using Tiles: For subtraction, Elaine owes $4 and she borrows $2 more from a friend. This may be represented as follows, using the zero principle.

1. Begin with ¯4.
2. Add extra zeros.
   This is still ¯4.
3. Remove +2
4. Answer
   ¯4 – 2 = ¯6



Have students work in pairs. Ask them each to roll two dice of different colours. Assign negative to one colour and positive to the other, and write a number sentence for the sum. Have them roll the two dice again, find the sum mentally and add the result to their previous score. Have them exchange turns until one person reaches +20 or ¯20. Ask why it would be fair to accept +20 or ¯20 as the winning score.

This activity can be modified and used with other operations. It can also be modified by assigning the negative or positive to specific colours after each roll, instead of maintaining the same designation throughout the game. Ask students if this change would allow them to get to +20 or ¯20 more quickly. Have them consider other possible rule changes, such as "You can't go over 20."

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❶ Information on Subtracting Integers is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

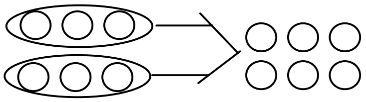
Teaching Notes

A similar game can be played using a deck of cards, where red represents one sign and black the other sign.

Multiplying Integers❶

Use tiles to show groupings; e.g.:

2 (– 3)



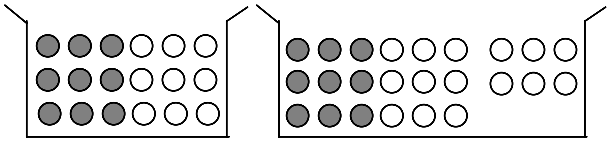
2 groups of $^{-}3$ gives $^{-}6$

Answer: $(2)(–3) = {}^{-}6$

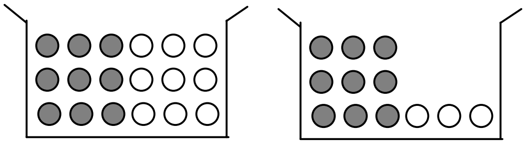
Addition of integers helps to establish some of the initial groundwork for multiplication of integers. Multiplication of integers should start with examining multiplication as repeated addition, as in:

4 sets of $(^{-}3) = (^{-}3) + (^{-}3) + (^{-}3) + (^{-}3)$.

The following is one way of using counters to model multiplication. Start with a container having an equal number of positives and negatives.



$+2\ (^{-}3)$ implies adding 2 sets of $^{-}3$. What is the total in the container?



$^{-}2\ (^{-}3)$ implies removing 2 sets of $^{-}3$. When 2 sets of $^{-}3$ are removed, what remains in the container?

When the first factor of the multiplication is positive, the operation is conceptualized as repeated addition. When the first factor of the multiplication is negative, the operation can be conceptualized as repeated subtraction.

To summarize in words, we say that we multiply the digits, and the sign of the answer can be determined as follows: same signs give a positive answer while different signs result in a negative answer.

Dividing Integers❶

Using Tiles: The following provides a starting point for modelling division.

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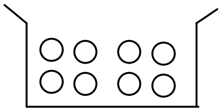
❶ Information on Multiplying Integers and Dividing Integers is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Start with an empty container.

¯8 ÷ (¯2)
Ask students how many groups of ¯2 are in ¯8.



Add successive groups of ¯2 to the container until there is ¯8 in it. Record the number of groups. Four groups of ¯2 were added to the container; therefore, ¯8 ÷ (¯2) = 4.

The model can be used with some modification for other division situations.

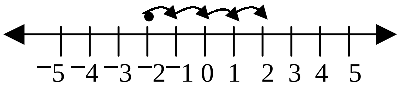
27 ÷ (¯9) =
¯60 ÷ (¯15) =

A scientific calculator will also perform these operations when the proper keying sequences are used.

Adding Integers

Using a Number Line: Start at the first number. Adding a positive indicates movement to the right; adding a negative indicates movement to the left; e.g.,

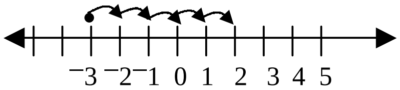
¯2 + 4



Therefore, ¯2 + 4 = 2.

Subtracting Integers

Using a number line requires the knowledge that subtraction is the opposite of addition. For example ¯3 – (¯5) = ¯3 + (+5) = +2



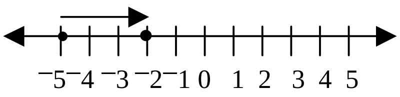
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

When using the number line to describe subtraction of integers, the use of the comparison model is more meaningful than the take-away model for subtraction. For example, $^{-}3 - (^{-}4)$ means how far is it from $^{-}4$ to $^{-}3$. The distance is 1. In going from $^{-}4$ to $^{-}3$, we move in a positive direction so the answer is +1. Note: When the second number of an addition or subtraction is negative, we use parentheses, such as: $^{-}4 + (^{-}3)$.

Subtraction may be easier for some students, by using a missing addend approach. For example, to find $^{-}8 - (^{-}4)$, ask: What would you add to $^{-}4$ to get $^{-}8$?

Show $^{-}2 - (^{-}5)$ on a number line.



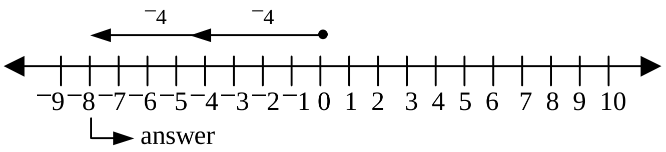
Start with $^{-}5$.
Ask how far it is to $^{-}2$, and in what direction.
Answer is 3.

Multiplying Integers❶

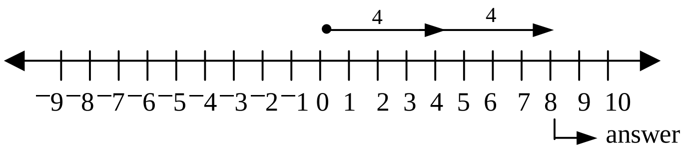
Using a Number Line: Net worth can also be used as a context for multiplication. Consider, for example, the impact on net worth if a person owes \$6 to each of three friends, or if a debt of \$6 to each of three friends is forgiven.

The number line can also be used to model problems such as:

$(2) \times (^{-}4) \rightarrow$ 2 sets of $^{-}4$



$(2) \times (+4) \rightarrow$ 2 sets of 4



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❶ Information on Multiplying Integers is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Patterning can then be used to justify the result for the multiplication of a negative by a negative.

(3)(¯2) = ¯6
(2)(¯2) = ¯4
(1)(¯2) = ¯2
(0)(¯2) = 0
(¯1)(¯2) = 2
(¯2)(¯2) = ?
(¯3)(¯2) = ?

Dividing Integers❶

Patterning is also useful in division; for example,

6 ÷ 2 = 3	¯6 ÷ (¯2) = 3
4 ÷ 2 = 2	¯4 ÷ (¯2) = 2
2 ÷ 2 = 1	–2 ÷ (¯2) = 1
0 ÷ 2 = 0	0 ÷ (¯2) = 0
¯2 ÷ 2 = ?	2 ÷ (¯2) = ?
¯4 ÷ 2 = ?	4 ÷ (¯2) = ?

Comparison of multiplication and division situations can also be very useful in helping students understand division of integers. After multiplication has been fully developed, the fact that multiplication and division are inverse operations can be utilized. For example, since ¯4 × 3 = ¯12, it must be true that the product divided by either factor should equal the other factor; therefore, ¯12 ÷ (¯4) = 3 and ¯12 ÷ 3 = ¯4. Likewise, if ¯4 × (¯3) = 12, then 12 ÷ (¯4) = ¯3 and 12 ÷ (¯3) = ¯4.

Using a missing factor can also be useful. For example, in the case of ¯16 ÷ (¯4), ask: what multiplied by ¯4 gives ¯16?

The concept of net worth can be linked with division as well. Students can think of owing $12 when an equal amount is owed to each of three friends. Students can determine how much is owed to each friend.

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❶ Information on Dividing Integers is reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

Strand: Number (Number Operations)
Specific Outcome: 9. Perform arithmetic operations with integers concretely, pictorially and symbolically. [PS, V] (7–16)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Performance

1. Have students use a number line or coloured counters to explain:

   a. $^{-}3 + 8 = 5$
   b. $^{-}5 – 3 = ^{-}8$
   c. $^{-}4 – (^{-}6) = 2$
   d. $9 + (^{-}2) = 7$
   e. $6 – 4 = 2$
   f. $8 – (^{-}3)=11$

2. Fill in the grid of numbers below, and comment on the patterns you observe.

×	4	2	$^{-}2$	$^{-}4$	$^{-}6$
4	16				
2		4			
$^{-}2$					
$^{-}4$		$^{-}8$			
$^{-}6$					

Portfolio

1. Using a newspaper or the Internet, refer to worldwide resort temperatures. Record the highest and lowest temperature, and determine the difference.

2. Make a monthly budget for yourself. What items would be indicated by positive integers? Which would be indicated by negative integers?

Paper and Pencil ❶

1. Complete the following patterns:

$9 \div 3 = 3$
$6 \div 3 = 2$
$3 \div 3 = 1$
$0 \div 3 = 0$
___ $\div 3 = ^{-}1$
___ $\div 3 =$ ____
___ $\div$ ___ = ____

$^{-}9 \div (^{-}3) = 3$
$^{-}6 \div (^{-}3) = 2$
$^{-}3 \div (^{-}3) = 1$
$0 \div (^{-}3) = 0$
____ $\div (^{-}3) = ^{-}1$
____ $\div$ ___ = ____
____ $\div$ ___ = ____
____ $\div$ ___ = ____

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❶ Paper and Pencil questions 1 to 6 and 8 are reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

Strand: Number (Number Operations)
Specific Outcome: 9. Perform arithmetic operations with integers concretely, pictorially and symbolically. [PS, V] (7–16)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

2. Based on the patterns, write a conclusion that can be drawn about division of a positive integer by a positive integer, a negative by a negative, a positive by a negative, and a negative by a positive. Confirm your conclusion by creating similar patterns beginning with

a. $20 \div 4$	b. $^-20 \div (^-4)$
$16 \div 4$	$^-16 \div (^-4)$

3. Write a number sentence to model the following situation. Chris and his three friends together owe \$32. They agreed to share the debt equally. What is each person's share of the debt?

4. Write a division or multiplication sentence to solve each problem.
   a. $? \div (^-3) = {}^-9$
   b. $57 \div ? = {}^-3$

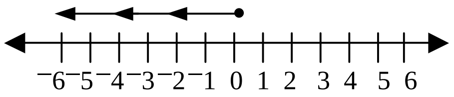
5. Write a number sentence for each of the following problems and use a diagram to model the situations.
   a. Fran lost 3 points in each round (hand) of cards that was played. If she played 4 rounds (hands), what was her score at the end of the game?
   b. Bill owed \$5 to each of 3 friends. What was his net worth, based on this situation?

6. Tell students that a list of temperatures for seven days was prepared, but the ink became wet and one temperature was unreadable. The mean temperature was $^-3$°C, and the six known temperatures were 2°C, $^-4$°C, $^-6$°C, $^-2$°C, 4°C, and $^-5$°C. Ask students to find the missing temperature.

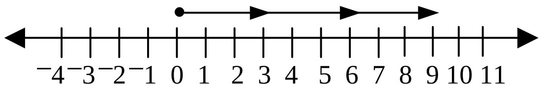
7. John saved \$50 during the fall. He owes \$15 to his friend. Because he had a good term report, his father gave him \$20. What is John's net worth?

8. Write number sentences for each of the following:

a.



b.



Teaching Notes

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

9. Solve:

   $^{-}12 + 14 =$

   $(^{-}3)(^{-}17) =$

   $^{-}60 \div 3 =$

   $42 - 70 =$

   $^{-}16 - (^{-}19) =$

Journal

1. a. Explain how adding and subtracting are related.
   b. Explain how multiplying and dividing are related.

2. Ask students if the following statement is true or false. The sum of a negative number and a positive number is always negative. Explain why or why not.

3. Tell students that a friend missed class the day that division of integers was first introduced. Ask them to write a detailed explanation for the friend to help him/her understand how to solve:
   a. $^{-}10 \div 5$
   b. $^{-}24 \div (^{-}6)$

Interview

1. Ask students to name as many pairs of integers as possible that have a product of $^{-}16$ and then a product of $^{+}16$. Ask what they notice about the number of possible pairs for the positive product versus the negative product.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

10. Illustrate and explain the order of operations. [PS, T, V] (7–17)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 77–78, 286–287
- *Mathpower 7*, pp. 98–99
- *Mathpower 9*, p. 44
- *Minds on Math 7*, pp. 132–135, 300–304
- *Minds on Math 8*, pp. 469–472
- *TLE 7*, Decimal Tile Explorer
- *TLE 7*, Exploring Decimals, Student Refresher pp. 20–21, Teacher’s Manual pp. 52–55
- *TLE 7*, Multiplication of Decimals, Student Refresher pp. 22–33, Teacher’s Manual pp. 56–59
- *TLE 7*, Division of Decimals, Student Refresher pp. 24–25, Teacher’s Manual pp. 60–63

Previously Authorized Resources

- *Journeys in Math 8*, pp. 22–23, 290–291
- *Journeys in Math 9*, pp. 12–13, 50–51, 74–75
- *Math Matters: Book 2*, pp. 6, 26

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes The most common error is $4 + 20 \div 2 \times 5 - 6 = 54$, where the students add the 4 + 20 first.	Introduce order of operations by giving an example, such as the following, where many answers can be derived. Example: Ask the students to find the answer $4 + 20 \div 2 \times 5 - 6$. Some typical answers could be $^{-}60$, 48, 0, $^{-}6$, 54. Ask the students to explain how they arrived at their answers. Since several answers can be logically explained, the students will see a need for order of operations. At this point, state the correct order in which operations are done with respect to the above example. • Do multiplication and division next, in the order in which they occur from left to right. • Do addition and subtraction last, in the order in which they occur from left to right.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Using the example and the various answers given at the beginning of the class, explain how each of the given answers could be correct if parentheses were inserted and the operations within the parentheses were done first. Students can then decide where to place the parentheses so that each answer could be correct; e.g.:

$(4 + 20) \div 2 \times 5 - 6 = 54$
$4 + 20 \div (2 \times 5) - 6 = 0$
$4 + 20 \div 2 \times (5 - 6) = {}^{-}6$

Exponents and order of operations should be dealt with next.

Students have difficulty with determining the base.

Ask the students to explain the difference between the following three expressions:

- ${}^{-}3 + ({}^{-}2)^2 \div 2$
- ${}^{-}3 + ({}^{-}2^2) \div 2$
- $({}^{-}3 + ({}^{-}2))^2 \div 2$

Order of Operations

- Operations within brackets are performed first.
- Operations with exponents are performed next.
- Multiplication and division are performed next, in the order they occur from left to right.
- Addition and subtraction are performed last, in the order they occur from left to right.

When using the calculator, keying sequences should be checked.

While some of these problems look complex, students need to break them down into smaller parts, solve the parts and calculate the answer.

Estimates are essential before students use their calculators.

Find the result:

a. $[{}^{-}27 + 7({}^{-}3)] \div [({}^{-}3) \times 2^2]$
b. $51 \div [({}^{-}6)^2 - (3^2 + 10)] + 23 - [16 - (4 \times 3)]$
c. $(6 + 2) \div ({}^{-}2 + 4)^2 \times (25 - 5 + 6 - 10)$
d. $\dfrac{(-5)^2 - [3(-7)]}{(-2)^2 - (9 - 3)}$

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Solve:
   a. $6 + (^-3)(^-22)$
   b. $^-7 + 2 \times {}^-14$
   c. $[^-12 + {}^-3] \times (^-16) + {}^-4$
2. Using brackets, write the expression so that it equals the given number.

a.	$2 + 3^2 \times 7 - 4$	33
b.	$72 \div 4 + 2^2 + 1$	8
c.	$10 + 5 + 3^3 \div 6$	7
d.	$10 - 7 - 2^2 \times 3$	$^-45$

3. Evaluate the following.
   a. $(17 - (^-4)) \times 2$
   b. $6^2 - 2^2 \div 4$
   c. $120 \div [18 - (^-1)^2 + 3]$
   d. $(^-3 + 4)(8 - 10) - (6 - 8)(7 - 5)$
   e. $(5 + 4) \div (8 - 7) + (16 + 4) \div (^-5)$
   f. $\dfrac{^-24 - 16 \times (^-2)}{6^2 \div 9}$
   g. $\dfrac{(^-6)(^-3) + (^-3)(4)}{(^-2)^2 + 8 \div 4}$

Portfolio

1. Collect contest skill-testing questions and solve them.
2. Use the digit 3 exactly 4 times separated by operations signs and/or parentheses to generate the numbers from 0 to 10; e.g.:
   $3 \times 3 - 3 \times 3 = 0$
   $(3 + 3 + 3) \div 3 = 3$

Performance①

1. Ask students to write a number sentence for the following and solve it, using the order of operations.
   a. Ms Jones bought the following for her project: 5 sheets of pressboard at \$8.95 a sheet, 20 planks at \$2.95 each, and 2 litres of paint at \$9.95. What was the total cost?

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① Performance questions 1 to 6 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

b. Three times the sum of $34.95 and $48.95 represents the total amount of Jim’s sales on April 29. When his expenses, which totalled $75.00, were subtracted, what was his profit?

c. Consider solving the number sentences for parts a and b by ignoring the order of operations. Would the solutions make sense in terms of the problems? Discuss.

2. Why is there an order of operations? Terry and Pat both answered a skill-testing question: $10 + 3 \times 2 - 1$. Terry got 15 and Pat got 25. Who was right and why?

3. Ask students to explain why it is necessary to know the order of operations to compute $4 \times 7 - 3 \times 6$. Ask them to compare the solution of this problem with the solution of $4 \times (7 - 3) \times 6$. Ask if the solutions are the same or different and why.

4. Owing to some faulty keys, the operation signs in these problems did not print. Use the information that is supplied to help determine which operators were used.
   a. (7.4 ☐ 2) ☐ 12.6 = 2.2
   b. 2 ☐ 7 ☐ 2 ☐ 3 = 13

5. Because the shift key of the keyboard did not work, none of the parentheses appeared in these problems. If the student has the right answer to both problems, identify where the parentheses must have been.
   a. $4 + 6 \times 8 - 3 = 77$
   b. $26 - 4 \times 4 - 2 = 12$

6. Billy had to answer the following skill-testing questions to win the contest prize. What are the winning answers?
   a. $234 \times 3 - 512 \div (2 \times 4)^2$
   b. $18 + 8 \times 2 - 32 \div 4$

   Billy was told that the correct answer for part b is 5, but Billy disagreed. What did the contest organizers do in solving the question that caused them to get 5 for the answer? Explain why you think they made that error.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

11. Add, subtract, multiply and divide fractions concretely, pictorially and symbolically. [E, PS, V] (8–9)

MANIPULATIVES

- Fraction circles
- Fraction strips

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 8*, pp. 88–123
- *Mathpower 8*, pp. 46–59
- *Minds on Math 8*, pp. 54–112
- *TLE 8*, Fractions Explorer
- *TLE 8*, Subtracting Fractions (unlike denominators), Student Refresher pp. 18–19, Teacher's Manual pp. 48–51
- *TLE 8*, Multiplying Fractions, Student Refresher pp. 20–21, Teacher's Manual pp. 52–55
- *TLE 8*, Dividing Fractions, Student Refresher pp. 22–23, Teacher's Manual pp. 56–59
- *TLE 8*, Exploring Fractions, Student Refresher pp. 12–13, Teacher's Manual pp. 36–39
- *TLE 8*, Adding and Subtracting Fractions, Student Refresher pp. 14–15, Teacher's Manual pp. 40–43
- *TLE 8*, Adding Fractions (unlike denominators), Student Refresher pp. 16–17, Teacher's Manual pp. 44–47

Previously Authorized Resources
- *Journeys in Math 8*, pp. 140–141, 146–151
- *Journeys in Math 9*, pp. 20–23

TECHNOLOGY CONNECTIONS

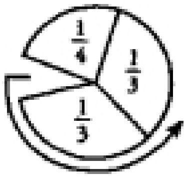
Strand: Number (Number Operations)
Specific Outcome: 11. Add, subtract, multiply and divide fractions concretely, pictorially and symbolically. [E, PS, V] (8–9)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should be able to perform simple calculations of fractions **without the use of a calculator**.

1. A brief review of the following concepts may be necessary: equivalent fractions, lowest terms and LCM. A number of manipulatives can be used to develop operations with fractions concretely, including circle models, pattern blocks, tangrams, money, number lines, fraction factory pieces, fraction bars and other fraction kits. Students should model with fractions; e.g.:



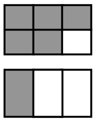
$\frac{1}{4}+\frac{1}{3}+\frac{1}{3}=\frac{11}{12}$

Since $\frac{1}{12}$ fits the gap, $\frac{11}{12}$ must be the sum.

$\frac{5}{6}-\frac{1}{3}$

Start with $\frac{5}{6}$

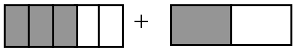
Represent $\frac{1}{3}$



Place the $\frac{1}{3}$ piece over the $\frac{5}{6}$ and $\frac{3}{6}$ are left.

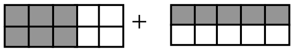
When moving toward developing an algorithm for addition and subtraction of fractions, a major focus should be placed on the writing of equivalent fractions. Once students internalize the fact that fractions can be added or subtracted symbolically when they reflect equal subdivisions of a quantity, they become less reliant on concrete or pictorial models. At this level, students may work with fractions written in improper or mixed number forms; however, they should be aware that applications of fractions to algebra in senior high school more often involve fractions represented in improper form.

$\frac{3}{5}+\frac{1}{2}$



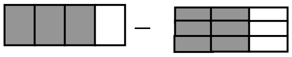
Represent both using the same subdivision of the whole.

$\frac{6}{10}+\frac{5}{10}=\frac{11}{10}$ or $1\frac{1}{10}$



Combine the two sets to produce a final answer.

$\frac{3}{4}-\frac{2}{3}$



Represent both using the same subdivision of the whole.

$\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$

Compare the two sets. The difference represents the final answer.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2. When any problem involving fractions is presented, it is important that students first attempt to solve it mentally. If it cannot be solved mentally, students should determine whether an estimate is sufficient or if an exact answer is required. The following are situations where mental computation would be expected:
    - When denominators are the same, or if the common denominator is easily determined; e.g., $\frac{1}{2}+\frac{1}{4}$, $\frac{7}{10}-\frac{1}{5}$. Such situations occur when one denominator is a multiple of the other.
    - When a simple fraction is subtracted from or added to a whole number; e.g., $2-\frac{1}{3}$, $4-\frac{2}{3}$, $3+4\frac{2}{3}$.

    Activities related to mental computation should generally be done for short periods of time. Five to ten minutes at the beginning of a class is usually sufficient.

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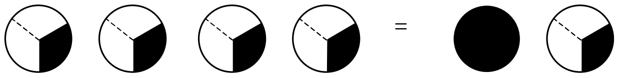
3. Multiplication of fractions should start with concrete and pictorial models but develop quickly to the symbolic level. Among the simpler combinations to model concretely or pictorially are:
    - a whole number by a fraction less than one; e.g., $4\times\frac{1}{3}$ uses repeated addition



($\frac{4}{3}$ or $1\frac{1}{3}$)

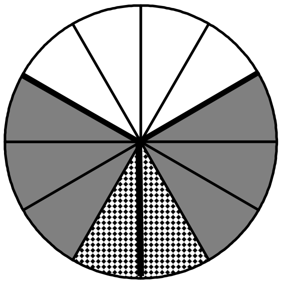
    - a fraction less than one by a whole number; e.g.,
      $\frac{1}{3}\times 6$. Think $\frac{1}{3}$ of 6
      Start with 6 objects. Divide into 3 groups.
      ○○○○○○ ○○ ○○ ○○
      How many are in each group?

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

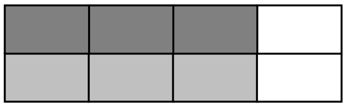
- a fraction less than one by any other fraction, especially when the numerator is 1; e.g.:

  $\frac{1}{4}$ of $\frac{2}{3}$
  Divide each $\frac{1}{3}$ into fourths.
  $\frac{1}{4}$ of $\frac{2}{3}$ makes
  $\frac{2}{12}$ or $\frac{1}{6}$.



It should be shown that "of" means multiplication. This may be done by comparing results in such examples as $\frac{1}{4}$ of 8 and $\frac{1}{4} \times 8$.

When work is done at the symbolic level, it should be supported by concrete or pictorial representations. Grid diagrams should also be considered when modelling multiplication. Students should notice that $\frac{1}{2}$ of $\frac{3}{4}$ is represented as $\frac{3}{8}$. By comparing the question with the result, students can start to speculate about a possible algorithm.



Modelling should always be related back to the symbols so that students make the connections clearly; otherwise, the use of models may not help support student understanding of the algorithms. Students should be able to work effectively with multiplication of fractions at the symbolic level in Mathematics Preparation 10.

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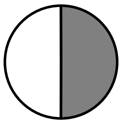
4. Students should be able to perform division of fractions at the symbolic level in Mathematics Preparation 10. Nevertheless, it is important that significant time be spent working with concrete and pictorial models. Initial examples for modelling should be chosen carefully and worked through by the teacher prior to instruction. These simpler examples should enable students to derive an algorithm. Situations that are well-suited to modelling with materials include:

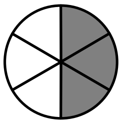
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

- a simple fraction divided by a whole number; e.g., for $\frac{1}{2} \div 3$, divide $\frac{1}{2}$ into 3 equal parts. What does each part represent?

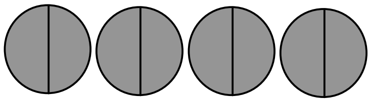


Divide $\frac{1}{2}$ into 3 equal parts.
Answer = $\frac{1}{6}$



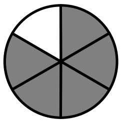
Divide into 3 equal parts.

- a whole number divided by a simple fraction; e.g., $4 \div \frac{1}{2}$ asks how many halves there are in 4



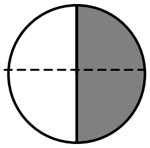
Count the number of halves in 4 objects. Since each object has two halves, $4 \times 2 = 8$. By comparing this to the original question, an algorithm can be developed.

- a simple fraction divided by a simple fraction, where the numerator of the divisor is one and both denominators are the same; e.g., $\frac{5}{6} \div \frac{1}{6}$ asks how many one-sixths there are in $\frac{5}{6}$

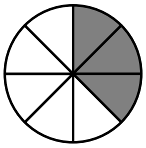


How many one-sixths are there in $\frac{5}{6}$?
Answer = 5

- a simple fraction divided by a simple fraction, where the numerator of the divisor is one and the fractions are compatible; e.g., $\frac{1}{2} \div \frac{1}{4}$ or $\frac{3}{8} \div \frac{1}{4}$.

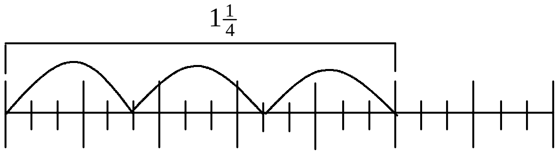


How many quarters are there in $\frac{1}{2}$?
Answer = 2



How many quarters are there in $\frac{3}{8}$?
Answer = $1\frac{1}{2}$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The number line can also provide a useful model for division. For example, suppose it takes $1\frac{1}{4}$ hours to do 3 chores. How long does it take for each chore, if they all require an equal amount of time? This can be modelled as follows:



Divide each quarter into 3 parts, which makes 15 parts in $1\frac{1}{4}$ hours. There will be 5 parts for each chore, and 12 parts in 1 hour, hence, $\frac{5}{12}$ hour for each chore.

There are two common algorithms for division that can be considered. The common-denominator algorithm involves finding a common denominator and dividing the numerators; e.g., $\frac{4}{3} \div \frac{1}{2} = \frac{8}{6} \div \frac{3}{6} = 8 \div 3 = 2\frac{2}{3}$. This can be modelled concretely using the process described in the fourth bullet above. The more traditional multiply by the reciprocal algorithm involves inverting the divisor and multiplying by it; e.g., $\frac{4}{3} \div \frac{1}{2} = \frac{4}{3} \times \frac{2}{1} = \frac{8}{3} = 2\frac{2}{3}$. It is necessary for students to understand the concept of reciprocal before they work with division using the multiply by the reciprocal algorithm. As a starting point for this algorithm, students can compare the solution of such problems as $8 \div \frac{1}{2}$ and 8×2.

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Strand: Number (Number Operations)
Specific Outcome: 11. Add, subtract, multiply and divide fractions concretely, pictorially and symbolically. [E, PS, V] (8–9)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

It is important for students to demonstrate their understanding of simple calculations of fractions **without the use of a calculator**.

Performance

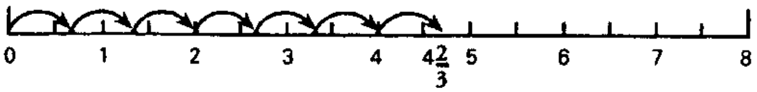
1. Show why the following is an incorrect procedure, through the use of concrete materials or diagrams.

   $\frac{3}{8}-\frac{1}{4}=\frac{3-1}{8-4}=\frac{2}{4}=\frac{1}{2}$

2. Draw a diagram to show why each of the following is true.

   a. $\frac{1}{3}\times 3=1$
   b. $6\times\frac{1}{3}=2$

3. What multiplication sentence is illustrated?



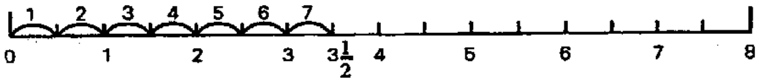
4. Place the numbers 1, 2, 3 and 4 in the boxes to get the smallest possible answer.

   $\frac{\square}{\square}\times\frac{\square}{\square}$

   Try the same type of problem, using the numbers 2, 3, 4 and 5. Choose a different set of four numbers and repeat the activity.

5. a. Use a diagram to explain why each of the following is true.
   $2\div\frac{1}{4}=8$
   $\frac{1}{2}\div 2=\frac{1}{4}$
   b. Compare the solutions in part a with the solutions to 2×4 and $\frac{1}{2}\times\frac{1}{2}$, and discuss any observations.

6. Write a division sentence for the following:



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Paper and Pencil❶

1. Create three pairs of fractions whose sum is $\frac{1}{2}$.
2. Create three addition and three subtraction sentences with the same result as $\frac{6}{12}+\frac{3}{12}$.

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❶ Paper and Pencil questions 5 to 7 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

3. A recipe requires $2\frac{1}{3}$ cups of flour to make 24 muffins. How many cups of flour would be required to make 60 muffins?
4. What might be the value of ☐, if $3\frac{1}{2} - 1\frac{2}{\square} < 2$?
5. Draw a diagram to show that $\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$.
6. Michael ordered three extra large pizzas and asked that each pizza be cut into sixteenths. If each person at Michael's party is likely to eat 3 pieces, how many people can the pizzas serve?
7. Lisa has $\frac{3}{4}$ of a large candy bar. She gave $\frac{1}{3}$ of what she had to Shannon.
   a. Explain, in at least two different ways, why Shannon received less than $\frac{1}{3}$ of what would have been a whole bar.
   b. What fraction of a whole candy bar does each girl have?

Interview

1. With the use of concrete materials, explain why the following is incorrect: $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$.
2. Ask students how they would convince someone that the following is incorrect: $\frac{5}{6} + \frac{5}{8} = \frac{10}{14}$.
3. Ask students:
   a. if an answer can be sixths when they add fourths and thirds, and to justify their response
   b. if an answer can be sevenths when they add fourths and thirds, and to justify their response.

Portfolio

1. Explain how to find the common denominator of two fractions if:
   a. one denominator is a multiple of the other
   b. the two denominators have a factor in common but one is not a multiple of the other.
2. Tell students that Frank works $7\frac{1}{4}$ hours a day for a five-day work week. He works 3 hours on Saturday and is paid time and a half.
   a. What weekly salary would he make at $9.25 per hour?
   b. If Frank has $95.00 per week taken out of his cheque for taxes and union dues, and his father makes him save $\frac{3}{5}$ of his take-home pay for university, how much spending money would be available to Frank on a weekly basis?

Strand: Number (Number Operations)
Specific Outcome: 11. Add, subtract, multiply and divide fractions concretely, pictorially and symbolically. [E, PS, V] (8–9)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

3. Write two fractional numbers that have a product between the given numbers for each of the following.
   a. 14 and 15
   b. $\frac{1}{3}$ and $\frac{1}{2}$

4. Complete the following patterns, and extend them for two extra lines. What pattern do you observe?

$9 \div 9 =$	$4 \div \frac{1}{2} =$
$9 \div 3 =$	$2 \div \frac{1}{2} =$
$9 \div 1 =$	$1 \div \frac{1}{2} =$
$9 \div \frac{1}{3} =$	$\frac{1}{2} \div \frac{1}{2} =$
$9 \div \frac{1}{9} =$	$\frac{1}{4} \div \frac{1}{2} =$
:	:

5. Caitlin decided to make muffins for the school cafeteria. Her recipe requires $2\frac{1}{4}$ cups of flour to make 12 muffins. Caitlin found there were exactly 18 cups of flour in the canister, so she decided to use all of it.

   a. Ask students how many muffins Caitlin can expect to get.
   b. The principal of the school liked Caitlin’s muffins and asked her to cater the school picnic next year. She will need to produce enough muffins for all 400 students. Ask students how many cups of flour Caitlin will require.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

12. Convert among fractions, decimals and percents to problem solve. [E, PS, T, R] (8–12)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 52–54, 104
- *Interactions 8*, pp. 134–154
- *Mathpower 7*, pp. 52–55, 150–161
- *Mathpower 8*, pp. 24–27, 116–117, 119–121, 126–130, 132–133, 138–139, 143
- *Minds on Math 7*, pp. 144–177
- *Minds on Math 8*, pp. 172–197
- *Minds on Math 9*, pp. 40–43
- *TLE 7*, Fraction Conversion, Student Refresher pp. 10–11, Teacher’s Manual pp. 32–35
- *TLE 7*, Decimal Conversion, Student Refresher pp. 12–13, Teacher’s Manual pp. 36–39
- *TLE 8*, Problem Solving with Fractions, Student Refresher pp. 24–25, Teacher’s Manual pp. 60–63

Previously Authorized Resources
- *Journeys in Math 8*, pp. 152–155, 198–203
- *Journeys in Math 9*, pp. 284–285
- *Math Matters: Book 2*, p. 28

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	• Final answers expressed as fractions should always be reduced to lowest terms. • Estimation of the answer should always be stressed. Students should be able to use the equivalent relationships of $\frac{1}{2}$ = 50%, $\frac{1}{4}$ = 25%, $\frac{1}{5}$ = 20% and $\frac{1}{10}$ = 10% to approximate other fractions and percentages. • Emphasize the importance of using conversions in a problem-solving context. Encourage students to relate their conversions to areas of personal interest, such as test marks, batting averages, local sales, sales taxes or deductions from pay cheques.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

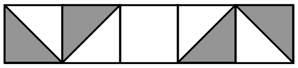
1. On Alice's last visit to the fitness centre, she spent 20% of her time in the pool, 15% on the stationary cycle, 25% in the racquetball court, 25% on the jogging track, and the remainder of her time in the locker room.
   a. What fraction of the time did she spend in the locker room?
   b. If Alice arrived at the fitness centre at 1:00 p.m. and left at 4:30 p.m., how many minutes did she spend in the pool?

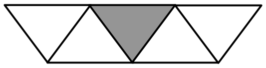
2. A ski shop offers an end-of-season clearance sale with the following reductions:

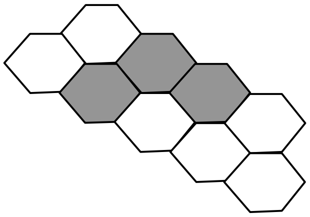
Day of the Sale	Percentage Reduction Off Original Price
1	10
2	20
3	25
4	30
5	45
6	50

   a. What fraction of the regular price is the sale price on the third day of the sale?
   b. If the percentage reduction was off the previous day's sale price, instead of the original price, approximately what fraction of the original price is the sale price on the third day?

3. Express the shaded region as a:
   a. fraction
   b. decimal
   c. per cent of the whole.







4. The hockey team at Centre Point High School won 12 of the 20 games it played.
   a. What percentage of the games did it win?
   b. If three of the games were tied, what percentage of the games did it lose?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

5. Tom's spending for the weekend is given in the following table.

Expense	Amount
Pizza	$15
Video	$4
Jeans	$60
Gasoline	$30
Swimming	$18
Hockey game	$25
Movie	$8

Express the amount of each expense as:

a. a fraction of the total (reduced to lowest terms)
b. a percentage of the total.

Journal Entry

1. Explain the steps for converting:
   - a fraction or decimal to a per cent
   - a per cent to a fraction or decimal
   - a decimal to a fraction.

Portfolio

1. Chart your daily schedule according to the amount of time, in hours, spent at each activity. Include at least seven activities, but limit the number of activities to no more than 10. Determine the time spent on each activity as a fraction of 24 hours. Then calculate the percentage of the day spent doing your two most time-consuming activities.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

13. Estimate and calculate operations, on rational numbers. [E, PS, T] (8–10)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 8*, pp. 198–215
- *Interactions 9*, pp. 87–90
- *Mathpower 8*, pp. 63–71, 74–75
- *Minds on Math 8*, pp. 283–295
- *Minds on Math 9*, pp. 32–36
- *TLE 8*, Adding and Subtracting Rational Numbers, Student Refresher pp. 26–27, Teacher's Manual pp. 64–67
- *TLE 9*, Rational Numbers and Problem Solving, Student Refresher pp. 20–21, Teacher's Manual pp. 52–55

Previously Authorized Resources

- *Journeys in Math 9*, pp. 68–75
- *Math Matters: Book 2*, p. 82

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Use consistent language when working with integers. Positive and negative refer to signed numbers, whereas plus and minus refer to operations.	Review Order of Operations • Operations within brackets are performed first. • Operations with exponents are performed next. • Multiplication and division are performed next, in the order they occur from left to right. • Addition and subtraction are performed last, in the order they occur from left to right. B rackets E xponents DM divide and/or multiply AS add and/or subtract This is a memory device for remembering order of operations. Discuss why there is a need for a specific order. Note that this is not always obvious but involves the need for consistency. Calculator keying must be checked.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Solve and discuss the answers to the following: a. $6 + ({}^-2) \times ({}^-4) \div ({}^-1)$ b. $[6 + ({}^-2)] \times ({}^-4) \div ({}^-1)$ c. $6 + ({}^-2) \times [({}^-4) \div ({}^-1)]$ 2. Solve: a. $6 + ({}^-3) \times ({}^-22)$ b. ${}^-7 +2 \times {}^-14$ c. $[{}^-12 + {}^-3] \times {}^-16+ {}^-4$ 3. Write an expression for each of the following, then solve. a. Multiply the sum of ${}^-35$ and 42 by ${}^-4$. b. Subtract ${}^-16$ from ${}^+18$, then divide by 4. c. Add the product of ${}^-7$ and ${}^-24$ to ${}^+52$. d. Divide ${}^+81$ by the product of ${}^-3$ and ${}^+3$. e. Subtract the sum of ${}^-42$ and ${}^-9$ from the product of ${}^-17$ and ${}^+5$. 4. Meadowview High put on a school dance. The admission charge was \$6.50 per person. The cost of the DJ was \$750.00, and the cost of decorating the gymnasium was \$200.00. If 180 people attended the dance, what was the profit?

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Portfolio

1. Have students use their calculators to answer the following question.
   Mary found the attendance reports for the first nine hockey games of the season to be 2787, 2683, 3319, 4009, 2993, 3419, 4108, 3539 and 4602. Tickets were sold for $12.75 each. The expenses for each game were as follows:

Stadium costs:	$15 000
Employee costs:	100 employees at $12.00 per hour for 6 hours per game
Miscellaneous costs:	$7000 per game

   What was the total profit for the nine games?

   Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7*.

2. Collect and solve contest skill-testing questions.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers. Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOMES

14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid.
    [PS] (5–13)
15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 214–215, 208–209
- *Interactions 8*, p. 108
- *Mathpower 7*, pp. 12–13
- *Mathpower 9*, pp. 88–89
- *Minds on Math 7*, pp. 316–320
- *TLE 7*, Order of Operations, Student Refresher pp. 26–29, Teacher’s Manual pp. 66–69
- *TLE 7*, Patterns and Relations, Student Refresher pp. 44–45, Teacher’s Manual pp. 100–103
- *TLE 9*, Rational Number and Problem Solving, Student Refresher pp. 20–21, Teacher’s Manual pp. 52–55

Previously Authorized Resources

- *Math Matters: Book 2*, p. 37 and integrated throughout the grades

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should be given the opportunity to work with rational numbers (positive and negative) in problem-solving situations involving real-life experiences relevant to their age group. Relate addition and subtraction of integers to real-life situations, such as football. If a team gains 5 yards and then loses 8 yards on the next play, what is the total gain/loss? Some background information on context, such as stock market or altitudes, may be required.

Strand: Number (Number Operations)

Specific Outcomes:
14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid. [PS] (5–13)
15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Example: Josée has $465.97 in her bank account. She writes cheques for $72.39, $16.95 and $173.00. Her GST rebate of $69.20 is automatically deposited in her account. What is her current balance?

$465.97 – $72.39 – $16.95 – $173.00 + $69.20 = $272.83

Her current balance is $272.83.

It is also important to stress the concept that there are many different methods of solving a problem and that all are acceptable as long as they are mathematically correct.

Obviously a farmer wouldn't really use this method of determining how many animals he has, but most students will enjoy solving the problem.

1. A farmer raises pigs and chickens. He has 15 animals in all, and the total number of legs of these animals is 46. How many of each does he have?

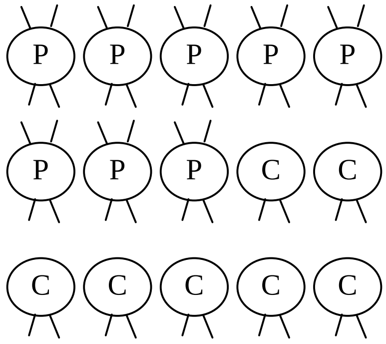
   Strategies:

   A. Use a Chart

# of Pigs	# of Chickens	Total # of Legs
5	15 – 5 = 10	5 × 4 + 10 × 2 = 40
7	15 – 7 = 8	7 × 4 + 8 × 2 = 44
8	15 – 8 = 7	8 × 4 + 7 × 2 = 46

   There are 8 pigs and 7 chickens.

   B. Diagram—Each circle represents an animal; each line segment represents a leg.
   Each animal has at least 2 legs; therefore, 30 legs in total if all chickens. Add 2 legs to each animal until the total is 46 legs.



INSTRUCTIONAL STRATEGIES/SUGGESTIONS

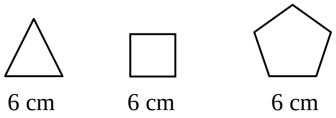
Teaching Notes

C. Algebra
Let p equal the number of pigs. Let $15 – p$ equal the number of chickens. Therefore:
$4p + 2(15 – p) =$ total number of legs
$4p + 30 – 2p = 46$
$2p = 16$
$p = 8$
There are 8 pigs and 7 chickens.

Understanding the problem should also include an understanding of the solution. Students should have some concept of a reasonable answer, and refer to this throughout the solving of the problem.

Supply a variety of manipulatives; e.g., grid paper, algebra tiles, cube-a-links and pattern blocks, and encourage students to model the problem using concrete materials if applicable.

2. The following figures were made with toothpicks that are 6 cm long.



a. Look at the pattern formed by the toothpick figures. How are the figures the same?
b. Sketch/create the next three figures in the pattern.
c. Calculate the perimeter of each figure and use this information to complete the table below:

number of sides	1	2	3	4	5	6	7	8
perimeter								

d. Graph the data from part c, using number of sides for the horizontal axis and perimeter for the vertical axis. Describe the pattern in the table and the graph.
e. Suppose this pattern continues. What is the perimeter of a polygon with 75 sides?

Strand: Number (Number Operations)
Specific Outcomes: 14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid. [PS] (5–13)
15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Performance

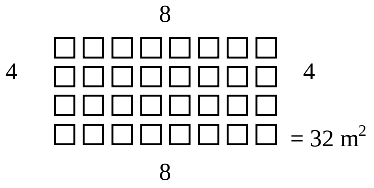
1. Lisa wants to build a rectangular dog run. What is the maximum area that can be enclosed if she has 24 linear metres of chain link fence? (Restrict your answer to the use of whole numbers.)

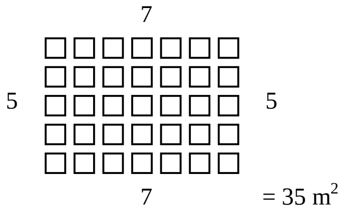
   a. Use a chart.

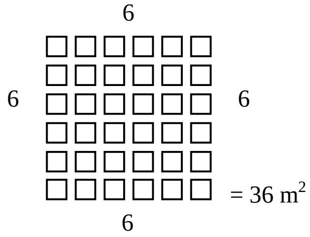
Width (m)	Length (m)	Area (m^2)
1	11	11
2	10	20
3	9	27
4	8	32
5	7	35
6	6	36
7	5	35

   The maximum area is 36 square metres.

   b. Use grid paper or unit algebra tiles. Each 1 × 1 square represents 1 square metre.
   - Cut the paper into 1 × 1 squares, then rearrange into rectangles—20 squares will be needed for the perimeter of the rectangles, since corner squares are counted for both length and width.
   - The rectangles are filled in with 1 × 1 squares, and the total number of squares in the rectangle will represent the area. The maximum area can then be determined.







Strand: Number (Number Operations)
Specific Outcomes: 14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid. [PS] (5–13)
15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

2. Students require grid paper and three different-coloured pencils. They are asked to outline a square with side lengths of at least four squares. Then have students colour the squares according to the following criteria.
   Yellow – all sides adjacent to another square
   Green – 3 sides adjacent to another square
   Blue – 2 sides adjacent to another square
   Example:

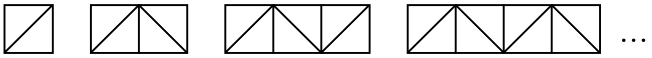
B	G	B
G	Y	G
B	G	B

Encourage students to use different lengths for the side of the square.

   a. What colour are the interior squares?
   b. Use words to describe the location of blue and green squares.
   c. Write a rule for the number of yellow, green or blue squares, given the length of the square as “*n*.”
   d. Is there a square that contains 250 green squares? 252 green squares? Explain.
   e. Is there a square with 400 yellow squares? 1000 yellow squares? Explain.
   f. Is it possible to have a square with eight blue squares?

Paper and Pencil

1. Each of the following rectangles is divided into squares, which in turn are divided into two triangles.



   a. Complete the table according to the pattern illustrated above.

Length of Rectangle	1	2	3	4	5	6
Number of Triangles	2	5		11		

   b. How many triangles would there be in a rectangle of length 20? Length 100?

2. Cathy earned and saved $1000 during the summer holidays. She does not plan on working during the school year. She budgets to spend $40 per week.
   a. Copy and complete the table below, assuming that Cathy sticks to her budget.
   b. Draw a graph to show how much Cathy has left at the end of each week.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

c. Write an equation that describes the amount of money she has left after t weeks of spending.
d. When will Cathy run out of money?

Week	Remaining Money
1	$960
2	$920
3	
6	
10	
	$480
	$320
	$200

3. Mount Everest is 8 850 m above sea level. Bernardin Cave in France is 10 344 m lower. What is the altitude of Bernardin Cave?

4. Land makes up three-tenths of the world's surface. One third of the land was once forest. What fraction of the world was once forest?

5. The value of Videoquest stock on Monday of a certain week is given in the table, along with the daily change for the week. Copy and complete the table.

	Opening Value	Change	Closing Value
Monday	10.00	–1.05	
Tuesday		+0.80	
Wednesday		+0.30	
Thursday		–0.45	
Friday		–0.25	

a. What is the closing value of the stock on Friday?
b. What would it cost to purchase 100 shares at opening time on Tuesday?
c. How much profit (or loss) would be realized if 200 shares were purchased at opening time on Monday and then sold at closing time on Thursday?
d. When during the week would it have been best to purchase stock?

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME

Illustrate and apply the concepts of rates, ratios, percentages and proportion to solve problems.

SPECIFIC OUTCOME

16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12)

MANIPULATIVES

- Base-ten blocks

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 152–174
- *Interactions 8*, pp. 125–163
- *Mathpower 7*, pp. 150–161, 174–189
- *Mathpower 8*, pp. 79–109
- *Minds on Math 7*, pp. 146–173, 412–441
- *Minds on Math 8*, pp. 118–167, 174–193
- *Minds on Math 9*, pp. 40–43
- *TLE 7*, Rate and Ratio, Student Refresher pp. 38–39, Teacher's Manual pp. 88–91
- *TLE 7*, Percent, Student Refresher pp. 36–37, Teacher's Manual pp. 84–87

Previously Authorized Resources

- *Journeys in Math 8*, pp. 168–186, 198–222
- *Journeys in Math 9*, pp. 268–294
- *Math Matters: Book 2*, pp. 200–219

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes The focus should be on an intuitive understanding of per cent.	**Per Cent❶** Number sense for per cent should be developed through the use of benchmarks: • 99% is almost all • 49% is almost half • 10% is not very much • 1% is very small in relation to total.

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❶ Information on Per Cent is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7* and *Atlantic Canada Mathematics Curriculum: Grade 8*.

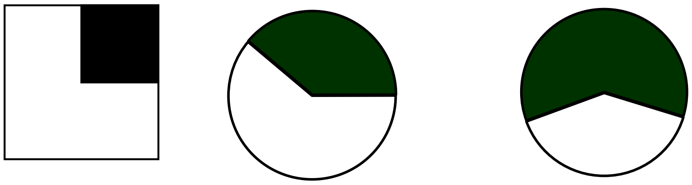
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Discussion should also focus on the contexts in which 1% would be considered high and the contexts in which 99% would be considered low. For example, consider the mercury content of fish that might be hazardous to humans versus the success rate of an air traffic controller.

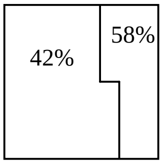
Visual representations should not be restricted to circular form.

- Students should relate to per cent visually. Ask students to estimate the per cent that the shaded portions of the following diagrams represent.

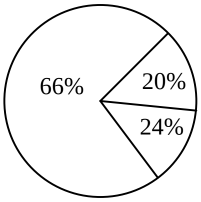


Ask students what is incorrect about each of the following:

a.



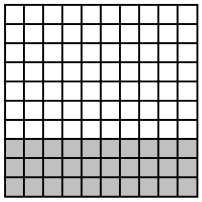
b.



c. Sarah wrote a test and had 8 questions right and 10 questions wrong. Sarah announced, 8 out of 10 is 80%—that's not a bad grade!

Students should be able to determine both accurate and approximate per cents. That is, students should be able to give an accurate value for the percentage shaded in diagram A below; and they should be able to estimate the percentage shaded in diagram B.

A.



B.

Students should make immediate connections between certain percentages and their fraction equivalents; e.g., 25%, 50%, 75%, and 20%, 30%, 40%. They should also be encouraged to recognize that per cents such as 51% or 49% are close to $\frac{1}{2}$ and, therefore, use $\frac{1}{2}$ for estimation purposes.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Complete the table.

Percentage	Fraction
	1/5
	1/3
	1/4
	1/2
	2/3
	2/5

When exact answers are required, students should be able to employ a variety of strategies in calculating per cent of a number, including:

- changing per cent to a decimal and multiplying; e.g.:
  12% of 80 = 0.12 × 80 = ?
- changing to a fraction and dividing; e.g.:
  25% of 60
  $= \frac{1}{4} \times 60$
  $= 60 \div 4$
- using the per cent key on a calculator.

Another technique that can be used to help students convert fractions to per cent, or per cent to fractions is to remember

$$\boxed{\frac{\text{part}}{\text{whole}}} \begin{matrix} \leftarrow \text{numerator} \\ \leftarrow \text{denominator} \end{matrix}$$

or $\boxed{\frac{\text{is}}{\text{of}}}$.

Example

1. 12% of what number is 17?

$$\begin{matrix} \text{part} & \rightarrow \\ \text{whole} & \rightarrow \end{matrix} \frac{12}{100} = \frac{17}{x} \begin{matrix} \leftarrow \text{is} \\ \leftarrow \text{of} \end{matrix}$$

2. What per cent of 25 is 8?

$$\frac{x}{100} = \frac{8}{25} \begin{matrix} \leftarrow \text{is} \\ \leftarrow \text{of} \end{matrix}$$

Evaluate:
18% of 40
25% of 320
40% of 16
90% of 200

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should be able to compute 7%, 10%, 25%, $33\frac{1}{3}$%, 50% and 1% mentally. This skill can be used to help with other mental computations.

Practice should also include per cents less than one and greater than 100.

Students will sometimes encounter occasions involving two percentages. Students should realize that, when combining percentages, they cannot add the percentages directly; e.g., if a tennis racket was already on sale for 20% off and the store announced a sale that read, "30% off all items in the store, including items already on sale," students should explore what happens when the two percentages are combined—compare calculating a 50% discount, versus taking off 20% followed by 30%.

Students should also be made aware that problems involving a discount can be solved in more than one way. For example, the discounted price can be determined either by finding 20% of the original price and subtracting or, more efficiently, by finding 80% of the original price.

In general, percentage increase or decrease is found using the following:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$$

Students have worked with per cent in previous grades. Per cents greater than 100, however, can be somewhat abstract for many students.

An increase of 25% means that the final is 125% of the original; e.g., after a 200% increase, a $50 item has a price of:
$50 + 200% of 50
= $50 + 100
= $150

Questions

1. Two-eighths of the tickets were sold for a concert held in a concert hall that has a 925-seat capacity. What percentage of the total capacity was used?

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2. The manager of a concert hall indicated that, in order to make a profit, the hall must be filled to at least 70% capacity or else the price of each ticket increases. The seating capacity is 1200, and advance ticket sales are at 912. Will a profit be made based on the number of tickets sold in advance sales?

3. A $400 set of tires is marked up 15%, and 7% GST is added to the marked up price. What is the selling price, including tax?

4. While there are many forms that can be used to express most numbers, certain forms are associated with various contexts or situations. Ask students which form is typically associated with each of the following:
   a. a special end-of-season sale at a clothing store (percentage)
   b. the batting average of a baseball player (decimal)
   c. the part of a cup that is used in a typical recipe (fraction)
   d. the teeth on the wheels and the gears in a bicycle (ratio)
   e. the sales tax (percentage)
   f. test scores (percentage)

5. Ask students to estimate a per cent that is a close approximation for each of the following and to indicate why their estimate is larger or smaller than the exact value. (They do not need to find the exact value to do this.)
   a. $\frac{7}{11}$
   b. 4:9
   c. $\frac{6}{13}$
   d. 7:16

6. Complete the following table. Assume Provincial Sales Tax (PST) is 6% and GST is 7%.

Item	Price	PST	GST	Total Cost
motorbike	$1480.00			
shoes	$89.99			
CD player	$124.50			

7. Have students work in pairs. Each student first works individually to create three problems, using a newspaper flyer. This student then solves these problems on a separate sheet of paper. Partners swap problems and solve them. Solutions are checked by the person who originally created the problems. When differences in solutions occur, both students work together to try to determine the source of error. (Sometimes the source of error may be a vaguely worded question. This can provide some information for further discussion with the small group or the whole class.)

Strand: Number (Number Operations)
Specific Outcome: 16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Mental Mathematics

1. If 2% of a certain number is 8:
   a. what would 10% of the number be?
   b. what is the number?

2. Sarah has a savings account that earns $\frac{1}{2}$% interest monthly. Jane has a savings account where she earns $5\frac{3}{4}$% annually. Who do you think would have more money in the bank at the end of one year if they both start with the same amount? Why?

Paper and Pencil❶

1. Mr. Jones bought a mining stock at $35. Two weeks later he sold it for $105. What was the percentage increase?

2. The Canadian dollar was valued at 70.0¢ US on Friday. On Monday, the opening value was 68.5¢ US. What was the percentage decrease?

3. Mack's Sound Emporium purchased CD players for $129 per unit and is planning to sell them for $195.99. It purchased 150 units.
   a. What is the percentage increase (markup) per unit?
   b. How much can Mack expect to make if he sells all the units?
   c. After four weeks, Mack realizes that the CD players are not selling as fast as he hoped, so he puts them on sale for 20% off. If he sells 56 units for the duration of the sale, how much money will he make on the items sold?

4. John's father said, "In my youth I could buy a chocolate bar and a soft drink for 20¢." What would be a typical cost for these items today? Estimate the percentage increase this represents.

5. A politician was elected with 2145 votes at a convention. If she received 58% of the votes cast, about how many votes were cast? (The solution mentally might be as follows: 60% of ☐ = 2100; guess 3000; 60% of 3000 = 1800; guess 4000; 60% of 4000 = 2400. Since 2100 is exactly halfway between the two guesses, a third guess might be 3500. Since an exact answer is not required, this seems like a reasonable estimate.)

6. Suits selling regularly for $185.00 were marked down by 25 per cent. To further improve sales, the discount price was reduced by another 15 per cent. What was the final selling price? What was the total per cent discount on the original price?

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❶ Paper and Pencil questions 1 to 3 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

This question assumes students have a knowledge of circle graphs and that 360° = 1 circle.

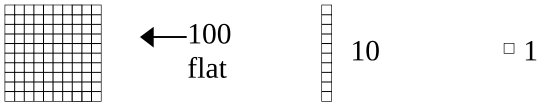
7. In reading a circle graph, Sarah realized that the sections of the graph did not contain any numbers or per cents. She decided to use the angle measures to help read it. The section that represented the number of red cars looked as if it was an angle of about 90°.
   a. Explain how to find what percentage of the cars are red.
   b. It was more difficult to estimate the number of degrees the blue cars represented, so Sarah used a protractor and found the angle to be exactly 145°. What percentage of the cars are blue?
   c. Suppose the circle graph represents the colours of the cars that pass an intersection during a one-hour period. Based on the information provided, if 400 cars passed this intersection, how many would you expect to be blue and how many would you expect to be red? How many would you expect to be neither blue nor red?

8. A store has a NO GST sale. Darcy purchased a skirt priced at \$39.99. When she paid for it, the clerk first subtracted 7% to get a new price and then added 7% GST to this new price. Is this a fair way to calculate the price? Why would a store use this practice?

9. McDunphy’s Burger Heaven has a sale on hamburgers. A hamburger is half price when you buy a medium drink and a medium fries. The regular prices are as follows: hamburger \$2.30, medium drink \$1.29 and medium fries \$1.39. What is the actual percentage off the regular price when you take into account what must be purchased to take advantage of the sale?

Performance

1. Tell students that a flat from a set of base-ten blocks represents 100% of something. Ask them to use base-ten blocks to represent
   a. 110%
   b. 125%
   c. 200%
   d. 450%



Strand: Number (Number Operations)
Specific Outcome: 16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Portfolio

1. Ask students to draw a rectangle and a triangle of any dimensions and to solve the following.
   a. Find the area and the perimeter of each figure.
   b. Increase the dimensions of each figure by 30%, and find the new perimeter and area.
   c. Decrease the dimensions of each of the original figures by 40%, and find the new perimeter and area.
   d. Find the ratio of new perimeter to original perimeter and the ratio of new area to original area for each of parts b and c. What do you notice?

   Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

2. Tell students that Sarah found out that the new car she bought would depreciate in value by 20% per year. Sarah paid $20 000 for the car and planned to keep it for three years. She wanted to find the car’s value at the end of three years and asked a friend to help. They decided to do their calculations independently and then compare answers. Sarah’s answer was $10 240, but her friend’s answer was $8000.
   a. Ask students how each of the answers was obtained.
   b. Ask them who they think is correct and to explain their choice.

Interview

1. Ask students to:
   a. explain why 70% is not a good estimate for 35 out of 80
   b. explain how to estimate the percentage when a test score is 26 correct out of 55
   c. change each of the following to a per cent, mentally, and explain their thinking:
   $\frac{2}{5}, \frac{4}{25}, \frac{6}{50}, \frac{7}{20}, \frac{1}{3}$
   d. estimate the per cent for each of the following, and explain their thinking:
   $\frac{7}{48}, \frac{5}{19}, \frac{7}{26}$
   e. indicate what per cent of a book is left to read if they have read 60 out of 150 pages, and ask them to explain their thinking.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

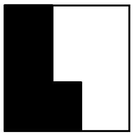
Journal Entry

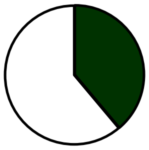
1. A certain chemical is dangerous to humans if it is found in the water supply at more than 275 parts per million.
    a. Ask students what per cent this represents.
    b. Ask them what per cent of the chemical would represent a danger level in 1000 L of water.

2. Ask students to critique the following situation and to explain why the reasoning is flawed.

   Jim found out that on his test the ratio of questions answered correctly to questions answered incorrectly was 12:13. He concluded that he should get a very good grade.

3. Estimate the percentage that is shaded for each of the following diagrams. Explain your reasoning.





STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME

Illustrate and apply the concepts of rates, ratios, percentages and proportion to solve problems.

SPECIFIC OUTCOMES

16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12)
17. Express rates and ratios in equivalent forms. [CN, PS, R] (8–15)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 152–174
- *Interactions 8*, pp. 125–163
- *Mathpower 7*, pp. 150–161, 174–189
- *Mathpower 8*, pp. 79–109
- *Minds on Math 7*, pp. 146–173, 412–441
- *Minds on Math 8*, pp. 118–167, 174–193
- *Minds on Math 9*, pp. 40–43
- *TLE 7*, Rate and Ratio, Student Refresher pp. 38–39, Teacher's Manual pp. 88–91
- *TLE 7*, Percent, Student Refresher pp. 36–37, Teacher's Manual pp. 84–87

Previously Authorized Resources

- *Journeys in Math 8*, pp. 168–186, 198–222
- *Journeys in Math 9*, pp. 268–294
- *Math Matters: Book 2*, pp. 200–219

TECHNOLOGY CONNECTIONS

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should understand that proportion is a statement of equality between two ratios. The emphasis should be on developing proportional reasoning. This topic is rich in real-world connections. It is when proportions are embedded in such real-life contexts as scale models, altering recipes and comparison shopping that students relate to the process used in finding unknown values.

The study of scale is an important application of work with ratio and proportion, and connects well with geometry and enlargements and reductions.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

Proportion

A *proportion* is a statement of equality of two ratios.

Example:

- The florist has a special on bouquets, 1 rose to 2 carnations. You want to buy one of these bouquets for your mother. If you want 6 roses in the bouquet, how many carnations will it include?

To solve the question of the flowers, students would set up a proportion statement such as the following:

Rose : Carnations	Roses : Carnations
1 : 2	6 : $\boxed{?}$

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should be very secure in using this type of notation before they are introduced to the fraction symbol for proportion.

Although proportions and equivalent fractions appear to be the same thing, they are not. Equivalent fractions are different symbols for the same amount. If you colour $\frac{1}{2}$ a piece of paper and fold it so that it now shows $\frac{2}{4}$, you still have the same amount coloured $\left(\frac{1}{2}=\frac{2}{4}\right)$. On the other hand, if you buy 2 bouquets of flowers and one has 1 daisy and 2 roses and the other has 2 daisies and 4 roses, the total number of flowers is different but the ratio of daisies to roses is the same (1 : 2 = 2 : 4) and therefore proportional.

Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

Sample Questions

1. If stereo speakers are to have good acoustics, the ratio of their depth to their width to their height is 1 to 2 to 3. If a speaker is 90 cm high, how deep and wide should it be to have good acoustics?

   Because 3 was multiplied by 30 to get 90, multiply the other terms by 30.

   $$\begin{array}{ccccc} \text{depth} & : & \text{width} & : & \text{height} \\ 1 & : & 2 & : & 3 \\ \downarrow & & \downarrow & & \downarrow \times 30 \\ 30 & : & 60 & : & 90 \end{array}$$

   The speaker should be 30 cm deep and 60 cm wide.

2. The ratio of the length of an adult's small intestine to the length of the adult's large intestine is about 4 to 1. If a person has a total of 7.5 m in small and large intestines, how long is the small intestine? How long is the large intestine?
   Method 1
   ratio of small : large : total is 4 : 1 : 5
   ○:☐ : 7.5
   Since 5 × 1.5 = 7.5, multiply all values by 1.5 to get 6 : 1.5 : 7.5

   Method 2
   x = the length of the large intestine
   $4x$ = the length of the small intestine
   $4x + x = 7.5$
   $5x = 7.5$
   $x = 1.5$
   The large intestine is 1.5 m and the small is (4 × 1.5) or 6 m.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

3. Theoretically, the ratio of the height a person can jump on the moon to the height a person can jump on Earth is 6 to 1. If a pole-vaulter can jump 4.5 m on Earth, how high could she jump on the moon?
   Moon : Earth
   6 : 1
   ☐ : 4.5
   Multiply both values by 4.5 to get 6 : 1 = 27 : 4.5.
   She can jump 27 m on the moon.

4. A bag contains red marbles, white marbles and blue marbles. The ratio of the red marbles to white marbles to blue marbles is 2 to 3 to 4. If there are 12 blue marbles in the bag, how many red marbles are there? How many white marbles are there?

5. The amount of gold in jewelry is measured in karats (K). This measure is a ratio expressed as a single number. The second term is understood to be 24. For example, the mark of 10 K means that the ratio of the mass of gold in the jewelry to the total mass of the metals is 10 to 24.

   If a ring is marked 14K and the ring (without any stones) has a mass of 72 g, what is the mass of the gold in the ring?

6. Sterling silver is an alloy of silver and copper in the ratio of 37 to 3.

   If a sterling silver goblet has a mass of 800 g, what is the mass of silver in the goblet? What is the mass of copper in the goblet?

7. If it takes 2.2 hours to write a seven-page essay, how long might it take to write an 18-page essay?

8. If the scale of a map is 1 : 50 000, and two towns are located 9 cm apart on a map, what is the actual distance between the two towns?

Rate

Rate is a quotient used to compare two measures of **different** units; e.g., kilometres per hour.

This topic is rich in real-life, problem-solving opportunities, such as rate of pay, cost per unit and rate of travel.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Sample Questions 1. If a 3-pack of juice boxes costs \$1.09, what would 12 juice boxes cost? (Students may use 3 × 4 = 12, so the cost is \$1.09 × 4 = \$4.36.) 2. Find the cost of 18 chocolate bars, if 10 chocolate bars cost \$2.19. $\frac{\$2.19}{10} = \frac{x}{1}$ $\quad x = 21.9¢ \cong 22¢$ So the cost of 18 chocolate bars is 22 × 18 = \$3.96. 3. Ask students to find how long it would take to produce an 18-page essay if they can produce a five-page essay in 2.2 hours. Students might do the following: $\frac{2.2}{5} = \frac{x}{18}$ $\qquad \frac{2.2 \times 18}{5} = x$

Strand: Number (Number Operations)
Specific Outcomes: 16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12)
17. Express rates and ratios in equivalent forms. [CN, PS, R] (8–15)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil①

1. Study each of the proportions, and estimate which of *a*, *b*, *c* or *d* represents the largest value. Solve to verify your estimate.
   a. $\frac{3}{7} = \frac{a}{28}$
   b. $\frac{b}{9} = \frac{3}{4}$
   c. $\frac{5}{c} = \frac{15}{33}$
   d. $\frac{5}{8} = \frac{23}{d}$

2. Pat has three cats for every five dogs in her kennel.
   a. In September, Pat had 25 dogs. How many cats did she have?
   b. In January, Pat had 48 cats and dogs altogether. How many of Pat's animals were dogs?

3. What is the scale of a map if 7.2 cm in the map represents a distance of 1800 km?

4. In planning his across-Canada tour, Orville estimated he could bike from Victoria to Halifax, a distance of approximately 6050 km, in 57 days. Calculate his rate of travel to the nearest kilometre per day.

5. If it takes Hugh 3.5 hours to drive from home to Calgary, a distance of 280 km, what is his average speed in kilometres per hour?

6. Joe earns $420 in a 40-hour work week. Calculate his hourly wage.

Interview①

1. Tell students that when making lemonade Sue uses 5 scoops of powder for 6 cups of water, and Sarah uses 4 scoops of powder for 5 cups of water. Ask students the following:
   a. Are the situations proportional to each other? Explain why or why not.
   b. In which situation is it likely the lemonade will be more flavourful? What assumptions did you make?

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① Paper and Pencil questions 1 and 2, and Interview questions 1, 2 and 4 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. Ask students to discuss in their journals whether or not the following could be solved using a proportion: David is 6 years old and Ellen is 2 years old. How old will Ellen be when David is 12 years old? 3. Ask students to explain whether or not, in the proportion $3 : 8 = 17 : x$, x can be a whole number. 4. Ask students to explain why 1 : 20 000 000 is another way to describe the ratio of 1 cm representing 200 km on a map. **Portfolio** 1. A statue of Lord Strathcona was made from a model. The height of the model was 25 cm. Ask students to find the height, in metres, of the statue if it was made using a scale of 1:15. 2. What scale would have been used if a 90 m building is 15 cm tall in a diagram?

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME	Apply exponent laws to solve problems.
SPECIFIC OUTCOME	18. Use exponent laws to evaluate expressions with numerical bases. [PS, R, T] (9–9)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 9*, pp. 28–36
- *Mathpower 8*, pp. 4–7
- *Mathpower 9*, pp. 20–21, 26–29
- *Minds on Math 8*, pp. 458–461
- *Minds on Math 9*, pp. 276–290
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–19, Teacher's Manual pp. 44–51

Previously Authorized Resources
- *Journeys in Math 9*, pp. 96–101

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should first understand the meaning of powers; e.g., 2^3, 4^4. Instruction should be designed so students discover rules/relationships and verify their discoveries. Otherwise students memorize "rules" without understanding why they work. Development of the laws through easily worked examples, should be the emphasis. Examples 1. $2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$ $= 2^7$ This would prompt students to express the law for multiplication of powers with the same base. 2. Have students calculate $(5^2)^2$, $(2^2)^4$ and $(3^2)^3$, and compare the results with 5^4, 2^8 and 3^6 to establish the power of a power rule. 3. Have students calculate $3^4 \div 3^2$ and $4^5 \div 4^3$ as follows. $\frac{3 \times 3 \times 3 \times 3}{3 \times 3} = \frac{81}{9} = 9 = 3^2$; $\frac{4\times4\times4\times4\times4}{4\times4\times4} = \frac{4\times4}{1} = 16 = 4^2$ Students can generalize the results to form a rule for dividing powers with the same base. They can verify with other examples.

Strand: Number (Number Operations)
Specific Outcome: 18. Use exponent laws to evaluate expressions with numerical bases. [PS, R, T] (9–9)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Dale has a stereo system with an amplifier as well as a preamplifier. The preamplifier boosts the signal by a factor of 10^5, and the amplifier boosts the signal by a factor of 10^4. Calculate the factor by which the signal is boosted after it has passed through both amplifiers, if this factor is a product of the two individual factors.

2. It has been estimated that each galaxy contains 10^{11} stars. If there are about 10^{11} galaxies in the universe, approximately how many stars are there in total? Explain why. (You can express the number of stars as $10^{11} \times 10^{11}$ or $(10^{11})^2$.)

3. Ask students to calculate $(3 \times 2)^3$ two different ways.

4. Simplify:
   a. $6^7 \div 6^5$
   b. $(3^5)^2$
   c. $2^3 \times 3^2$
   d. -5^2
   e. $(\frac{2}{5})^3$
   f. $(\frac{2}{3})^{-2} \times (\frac{2}{3})^3$

Journal

1. Which is greater, 2^{-4} or 4^{-2}?
   Explain how you know.
   (You may use your calculator to confirm your answer.)

2. Solve the following question mentally, and explain your thinking.
   $3^3 \times 3^{-2} \times 4^3 \times 4^{-3} \times 2^4 \times 2^{-2}$

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME	Apply exponent laws to solve problems.
SPECIFIC OUTCOME	19. Understand and use the exponent laws to simplify expressions with variables as bases and use substitution to calculate a numerical value. [PS, R, T] (9–9)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 52–61
- *Interactions 9*, pp. 30–34, 38–42, 120–123
- *Mathpower 9*, pp. 20–43
- *Minds on Math 9*, pp. 292–299
- *TLE 9*, Laws of Exponents 1–3, Student Refresher pp. 10–15, Teacher's Manual pp. 32–43
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–19, Teacher's Manual pp. 44–51
- *TLE 9*, Simplifying and Evaluating Exponential Expressions, Student Refresher pp. 22–23, Teacher's Manual pp. 50–55

Previously Authorized Resources

- *Journeys in Math 9*, pp. 102–103, 132–133, 138–139
- *Math Matters: Book 2*, pp. 95–101

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Premature use of a calculator in work with exponent laws causes many students trouble. Provide opportunities for students to simplify expressions and to calculate a numerical value without the use of a calculator to ensure that process is understood.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Questions

1. Mentally, or with paper and pencil, calculate the value of each of the following expressions:
   - a. $\dfrac{1+2^2}{3^2-2^2}$
   - b. $5 \times 4^6 \div (4^2)^2$
   - c. $1 + 7n^2$, if $n = 3$
   - d. $(6^2)^3 \div (6^2)^2$
   - e. $\dfrac{6^2 \div 2^2 - 2}{5^2 - 14}$

Calculations with powers are done easily with a scientific calculator after students have developed an understanding of the exponent laws and can apply them to simple expressions.

Students often have trouble doing the keying sequence for fractions on the calculator as they omit the brackets; e.g.:

For $\left(\dfrac{4}{5}\right)^3$, students will often input $4 \textcircled{\div} 5 \textcircled{y^x} 3 \textcircled{=}$ which won't give the correct answer.

Sometimes it is easier to simplify within parentheses first; other times it is easier to apply the power law first; e.g.:

Evaluate $\left(\dfrac{6^3}{4^2}\right)^2$

Method 1: $\left(\dfrac{6^3}{4^2}\right)^2 = \dfrac{6^6}{4^4} = \dfrac{46656}{256} = 182.25$

Method 2: $\left(\dfrac{6^3}{4^2}\right)^2 = \left(\dfrac{216}{16}\right)^2 = (13.5)^2 = 182.25$

Students should be familiar with both methods.

2. Evaluate each of the following, using both methods of simplifying. Express the result as a whole number or fraction in lowest terms.
   - a. $(4^3)^2$
   - b. $\left(\dfrac{9^2}{3^3}\right)^{-2}$
   - c. $(4^3 \times 2^{-3})^2$

Strand: Number (Number Operations)
Specific Outcome: 18. Understand and use the exponent laws to simplify expressions with variables as bases and use substitution to calculate a numerical value. [PS, R, T] (9–9)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Keying sequences are not unique. Students should be encouraged to explain their own preferred keying sequence.

Paper and Pencil

1. Simplify, then evaluate each expression for $x = -2$.
   a. $x^3 \bullet x^2$
   b. $\frac{x^6}{x^3}$
   c. $(x^2)^3$
   d. $x^{-3} \bullet x^{-1}$
   e. $\frac{x^{-2}}{x^{-3}}$
   f. $(x^{-2})^{-1}$

2. Calculate $(1.6 \times 2.5)^3$ using your calculator. Describe and explain the keying sequence you used.

3. Evaluate the following, without using a calculator:
   a. $(5 - 2)^3 - 7$
   b. $\left(\frac{4^2 + 2}{2}\right)^2$
   c. $7^3 - 7^2 \times 2$

4. Evaluate the following, using a calculator:
   a. $(3.14 \times 7^2) \times (5.26 \times 4)^2$
   b. $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^3$
   c. $\frac{(9^3 \times 8^4)^2}{3^4}$
   d. $\left(\left(-\frac{1}{5}\right)^2\right)^3$

5. Use the rules of powers and exponents to find the value of n.
   a. $(5^n)^n = 625$
   b. $(4^n)^3 = 4^{-6}$
   c. $(n^2)^3 = 64$
   d. $n^5 \div n^3 = 36$