STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME

1. Generalize a pattern arising from a problem solving context using mathematical expressions and equations, and verify by substitution. [C, CN, PS, R] (8–1, 9–1)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 206–209
- *Interactions 8*, pp. 30–36, 222–226, 229–230
- *Interactions 9*, p. 96
- *Mathpower 7*, pp. 194–195, 202–205
- *Mathpower 8*, pp. 8–9, 146–151, 154–158
- *Mathpower 9*, pp. 58–66
- *Minds on Math 8*, pp. 350–351
- *TLE 7*, Patterns and Relations, Student Refresher pp. 44–45, Teacher's Manual pp. 100–103
- *TLE 9*, Logic, Problem Solving and Mathematical Modelling, Student Refresher pp. 26–29, Teacher's Manual pp. 64–71

Previously Authorized Resources

- *Journeys in Math 8*, pp. 266–267
- *Journeys in Math 9*, pp. 222–223

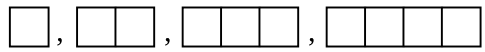
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should be able to look at a sequence of numbers or a diagram and describe the pattern displayed. They should be able to use mathematical expressions and equations to describe the pattern. 1. 6, 10, 14, 18, … a. Describe the pattern in the numbers above. b. Give the next three numbers in the pattern. Solution a. Each number is 4 greater than the previous one. b. 22, 26, 30

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2. Toothpicks are used to form a series of squares as shown.



a. Copy and complete the following table.

No. of squares	1	2	3	4	5	6	7
No. of toothpicks needed	4	7	10	?	?	?	?

b. Describe in words the relationship between the number of toothpicks needed and the number of squares being formed.
c. Write an equation that shows the relationship between the number of toothpicks needed and the number of squares being formed.
d. Use your equation from part c to determine the number of toothpicks needed to form a row of 20 squares.

Solution

a.

No. of squares	1	2	3	4	5	6	7
No. of toothpicks needed	4	7	10	13	16	19	22

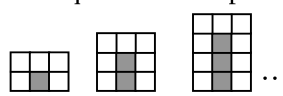
b. The number of toothpicks needed is one greater than three times the number of squares.
c. $t = 3s + 1$
d. $t = 3s + 1$
$t = 3(20) + 1$
$t = 61$
The number of toothpicks needed for 20 squares is 61.

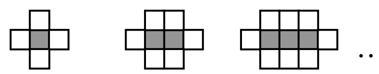
Strand: Patterns and Relations (Patterns)
Specific Outcome: 1. Generalize a pattern arising from a problem solving context using mathematical expressions and equations, and verify by substitution. [C, CN, PS, R] (8–1, 9–1)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**[1]

Paper and Pencil[1]

1. For each of the tile patterns:
   a. make a table of values, and describe the pattern in words
   b. use the pattern and description to write a mathematical equation identifying what the variable(s) represent
   c. use the equation to help determine the tenth entry in the table.

   i. 

   ii. 

2. A certain rectangle has a length that is $\frac{1}{2}$ of the width.
   a. Make a table showing the relationship between the width and the perimeter.
   b. Describe, in words, the relationship between the width and the perimeter.
   c. Write a mathematical rule to relate the width and perimeter, identifying what the variable(s) stand for.
   d. Use the rule to find the perimeter when the width is 99 metres.

[1] Paper and Pencil questions 1 and 2 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME

2. Given a first degree equation, substitute numbers for variables and graph and analyze the relation. [C, PS, R, V] (8–2)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 164–167, 186, 188, 223–233
- *Interactions 8*, pp. 227–228, 284–289
- *Mathpower 7*, pp. 212–217
- *Mathpower 8*, pp. 162–167
- *Minds on Math 8*, pp. 118–119, 355
- *TLE 9*, Mathematical Modelling, Student Refresher pp. 28–29, Teacher's Manual pp. 68–71

Previously Authorized Resources

- *Journeys in Math 8*, pp. 310–311
- *Journeys in Math 9*, pp. 310–321
- *Math Matters: Book 2*, p. 224

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The first-degree equations required in the course are of four types: 1. $y = mx$, with m positive 2. $y = mx + c$, with m a natural number 3. $y = mx + c$, with m a positive rational number 4. $y = mx + c$, with m negative The best procedure is to gather data, usually four or five data pairs, sufficient to determine the linear equation, and then use the equation to predict the value of y for a given value of x and also to determine the required value of x that is needed to yield a given value of y. It is recommended that the variable x be referred to as either the independent variable or the manipulated variable, and that the variable y be referred to as the dependent variable or the responding variable.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Examples Case 1 The independent variable could refer to the hours worked, *m* could refer to the hourly wage rate, and the dependent variable could refer to the gross wages. In this case, *y* and *x* could be replaced by *W* (wages) and *t* (time). Case 2 The independent variable could refer to the number of quarters added to a purse, *m* would equal 25, and the dependent variable could refer to the total money in the purse. In this case, *y*, *x* and *c* could be replaced by *F* (finishing money in cents), *n* (number of quarters added) and *S* (starting money in cents). Case 3 The independent variable could refer to the volume of a liquid in a beaker, *m* could refer to the density of the liquid, and the dependent variable could refer to the combined mass of the liquid and beaker. In this case, *y*, *x* and *c* could be replaced by *M* (total mass), *V* (volume of liquid) and *B* (mass of the empty beaker). Case 4 The independent variable could refer to the advertised cost of a pair of shoes, *m* would be fixed at –1.07 to allow for a 7% GST, and the dependent variable could refer to the change given from $100. In this case, *y*, *x* and *c* could be replaced by *C* (change), *S* (cost of a pair of shoes) and 100.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

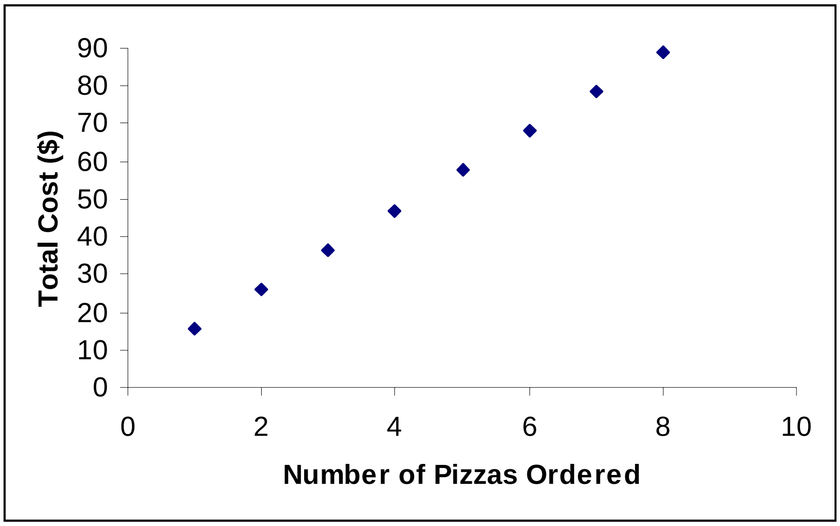
Teaching Notes

Paper and Pencil

1. Frank's Pizza House charges $5.00 for delivery and $10.50 for each small pizza ordered. The total cost can be represented by the equation $C = 5 + 10.5n$, where C is the total cost in dollars, and n is the number of small pizzas ordered.
   a. Use the equation to determine the total cost of seven small pizzas, including delivery.
   b. Use the equation to determine the total cost of four small pizzas, including delivery.
   c. Make a table of values for the cost of pizzas and delivery for one to eight small pizzas. Graph the data.

1. Solution
   a. $C = 5 + 10.5n$
      $C = 5 + 10.5(7)$
      $C = 78.5$
      The total cost is $78.50.
   b. $C = 5 + 10.5n$
      $C = 5 + 10.5(4)$
      $C = 47$
      The total cost is $47.00.
   c.

No. of pizzas	1	2	3	4	5	6	7	8
Total cost ($)	15.50	26.00	36.50	47.00	57.50	68.00	78.50	89.00



STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME 3. Translate between an oral or written expression and an equivalent algebraic expression. [C, CN] (8–3, 9–8)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 156–157, 276–278
- *Interactions 7*, pp. 210–215, 219–220
- *Interactions 8*, pp. 224–226, 239, 284, 286, 298–299
- *Interactions 9*, pp. 224–227
- *Mathpower 7*, pp. 198–199
- *Mathpower 8*, pp. 153, 174–175
- *Mathpower 9*, pp. 66–69
- *Minds on Math 7*, pp. 338–341
- *Minds on Math 8*, pp. 360–365, 472
- *Minds on Math 9*, pp. 139–143, 154–160
- *TLE 8*, Exploring Equations, Student Refresher pp. 42–43, Teacher's Manual pp. 96–99
- *TLE 9*, Mathematical Modelling, Student Refresher pp. 28–29, Teacher's Manual pp. 68–71

Previously Authorized Resources
- *Journeys in Math 8*, pp. 342–343
- *Journeys in Math 9*, pp. 126–127, 174–177
- *Math Matters: Book 2*, pp. 22, 78–79

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	In order to successfully solve problems using algebra, students need to make the connection between the written word and the corresponding algebraic expression. They must be able to identify the pertinent verbal expressions in the problem, and then translate those verbal expressions into algebraic expressions. 1. Write an algebraic equation or expression for the following: a. six more than a number cubed b. five less than a number c. the product of two numbers diminished by five d. four times a number decreased by six e. one quarter of a number is twelve f. six times a number is eighteen

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Solution a. $n^3 + 6$ b. $n - 5$ c. $xy - 5$ d. $4x - 6$ e. $\frac{1}{4}m = 12$ f. $6x = 18$ 2. Write a verbal expression for the following algebraic expressions. a. $3p - 4$ b. $m + 5$ c. $x^2 - 2 = 14$ d. $2t + 6 = 12$ 2. Solution a. four less than three times a number b. five more than a number c. two less than the square of a number is fourteen d. six more than twice a number is twelve 3. Write an equation or equations that could be used to solve the following problems. a. What are the dimensions of a rectangle that is twice as long as it is wide, and whose perimeter is 42 cm? b. Two numbers have a sum of 48. The larger number is four more than the smaller. What are the two numbers? c. Mary is 9 years older than Joan. The sum of their ages is 39. How old is Joan? 3. Solution a. $2(2w + w) = 42$ or $L = 2W$, $2L + 2W = 42$ b. $x + x + 4 = 48$ or $L + S = 48$, $L = S + 4$ c. $J + 9 + J = 39$ or $M = J + 9$, $M + J = 39$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. An airplane travels eight times as fast as a car. The sum of their speeds is 990 km/h. How fast is each one travelling? 2. John earned four times as much as Jerry. The difference in their earnings is $150.00. How much did Jerry earn? 3. A 23-metre rope is cut into three pieces. The first piece is twice as long as the second piece. The third piece is one metre shorter than the second piece. How long is each piece?

STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME 4. Write equivalent forms of algebraic expressions, or equations, with integral coefficients. [C, CN, R] (9–3)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 150–155
- *Interactions 9*, pp. 230–232
- *TLE 9*, Equivalent Expressions, Student Refresher pp. 30–31, Teacher's Manual pp. 72–75

Previously Authorized Resources

- *Journeys in Math 9*, pp. 172–173

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes This outcome is crucial for success in Grade 10, as a formula must be rearranged before it can be graphed or inputted into a spreadsheet document.	A formula is often not written in the form needed for a particular problem. For example, the formula for density is $D = \frac{M}{V}$, where M is mass and V is volume. If density and volume are known and mass is unknown, it may be useful to rewrite the formula with mass as the subject: $M = D \times V$. The known values for D and V could then be substituted and the unknown mass calculated. 1. Rewrite each formula to solve for the given variable. a. $A = lw$, w b. $P = 2l + 2w$, w c. $A = \frac{1}{2}bh$, b d. $C = 2\pi r$, r e. $E = mc^2$, c

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Solution a. $w = \frac{A}{l}$ b. $w = \frac{P - 2l}{2}$ c. $b = \frac{2A}{h}$ d. $r = \frac{C}{2\pi}$ e. $c = \sqrt{\frac{E}{m}}$ 2. The distance travelled by a car is given by the formula $d = vt$, where d is the distance in km, v is the speed (velocity) in km/h, and t is the time in hours. Rearrange this formula as needed to solve the following problems. a. How fast would a car have to travel to go 500 km in 4 hours? b. How long would it take a car to go 385 km at 110 km/h? c. How far would a car go in 5 hours at 95 km/h? 2. Solution a. $v = \frac{d}{t}$ $v = \frac{500}{4}$ $v = 125$ km/h b. $t = \frac{d}{v}$ $t = \frac{385}{110}$ $t = 3.5$ h c. $d = vt$ $d = (95)(5)$ $d = 475$ km

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Complete the table.

Distance	Velocity	Time
760 km		8 h
488 km	80 km/h	
	60 km/h	3 h
225 km		2.5 h
880 km	110 km/h	
	75 km/h	6.5 h

Journal or Interview

1. Explain why each of the following is incorrect.
   a. $3x = 3 + x$
   b. $-x^2 = (-x)^2$
   c. $x - 1 = 1 - x$
   d. $x^2 = 2x$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME 5. Identify constant terms, coefficients and variables in polynomial expressions. [C] (9–7)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 46, 72–75
- *Interactions 7*, pp. 210–211
- *Interactions 9*, pp. 104–105, 127
- *Mathpower 7*, p. 196
- *Mathpower 9*, pp. 16, 62
- *Minds on Math 9*, pp. 320–325
- *TLE 9*, Terms of Polynomials, Student Refresher pp. 42–43, Teacher's Manual pp. 96–99

Previously Authorized Resources

- *Journeys in Math 9*, p. 126
- *Math Matters: Book 2*, pp. 84–86

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should be familiar with the terminology but should not be required to memorize formal definitions. Provide opportunities for students to discuss mathematical concepts, and encourage proper usage of terminology in those discussions. Students should be able to give examples of constants, coefficients and variables. A polynomial is made up of several parts. Each of the groups of numbers and/or letters, separated by addition or subtraction signs, is called a term. Each term is made up of a number factor, called the coefficient, and a variable factor (or factors). A term that consists of a number by itself is called a constant. • A monomial consists of one term; e.g., $4n^2$. • A binomial consists of two terms; e.g., $4n^2 + 4mn$. • A trinomial consists of three terms; e.g., $4n^2 + 4mn + 8$.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Algebra tiles are cardboard squares and rectangles that are white on one side and coloured on the other. The "*x*" set consists of a number of unit tiles, (squares that are one unit in length), a number of *x* tiles (rectangles that are *x* units long and 1 unit wide), and a number of x^2 tiles (squares that are *x* units in length). The length of the *x* tile is not an integral number of 1tiles. Students should be encouraged to make this discovery. There is also a "*y*" set and an overhead set. The "*y*" set consists of *y* tiles (rectangles that are *y* units long and 1 unit wide), *xy* tiles (rectangles that are *x* units wide and *y* units long), and y^2 tiles (squares that are *y* units long). Again the *y* tile is not an integral number of unit tiles. For introductory work, the "*x*" set is probably adequate.

In the term $4n^2$:

- 4 is the coefficient
- *n* is the variable.

In the term $4mn$:

- 4 is the coefficient
- *m* and *n* are the variables.

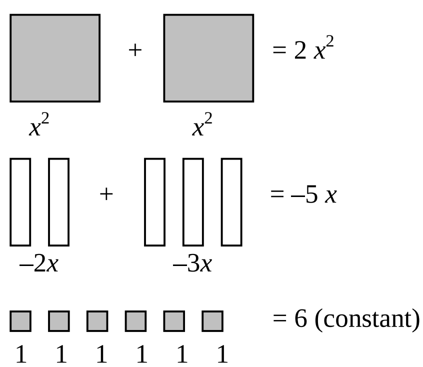
In the term 8:

- 8 is a constant.

Terms with identical variables including their exponents are called like terms. Examples of like terms are $3a$ and $2a$, n^2 and $-4n^2$, and a^2b and $-3a^2b$.

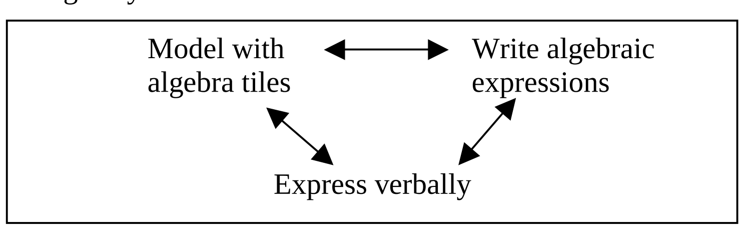
Collecting like terms can be demonstrated concretely using algebra tiles. These tiles can be purchased commercially or can be handmade.

Tiles that are the same shape (like terms) can be combined; e.g.:



Translation of Expressions

To express quantities, using symbols, models and words interchangeably:

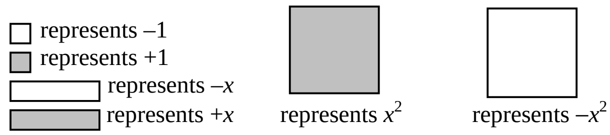


INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

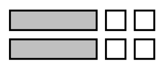
Resources that contain suggestions for the use of algebra tiles can be found in the *Kindergarten to Grade 9 Mathematics Resources: Annotated Bibliography*.

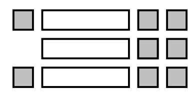
The following describes how the algebra tiles used in the activities in this manual will represent the variables and constants shown, unless stated otherwise.



Examples

1. Write an algebraic expression for the quantities illustrated by the algebra tiles.

   a. 

   b. 

2. Arrange algebra tiles to illustrate these algebraic expressions.
   a. $2x + 3$
   b. $-x - 2$

3. Express the following mathematical expressions verbally.
   a. xy
   b. $2x + 1$
   c. $-2(y + 3)$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	See tasks below

Paper and Pencil

1. What is the coefficient of $-6a^4b$? of $8a^2b^5$?

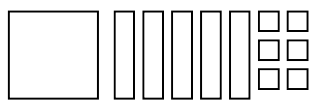
2. What are the constant terms in the expressions $4x - 3 + 2y$? $7x + 8y - 8$?

3. Write a trinomial involving two variables, having coefficients of 6 and –3, and a constant term of 2.

4. Classify each of the following according to the number of terms.
   a. $x^2 + 2$
   b. $x^2 + 2x + 5y$
   c. $3x^2$
   d. $xy + 7$

Performance

1. Given:



   a. What expression does this model represent?
   b. Identify the constant.
   c. Identify the coefficient of the:
      - x term
      - x^2 term.

Journal/Interview

1. State the difference between each item in the pair, and give an example of each:
   - a variable and a constant
   - a variable and a coefficient
   - like and unlike terms.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME 6. Evaluate polynomial expressions, given the values of the variables. [E] (9–8)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 216–217
- *Interactions 9*, pp. 106–109
- *Mathpower 7*, pp. 196–197
- *Mathpower 8*, pp. 152–153
- *Mathpower 9*, pp. 62–65
- *Minds on Math 7*, pp. 355, 359
- *Minds on Math 8*, pp. 352–360, 366–368
- *Minds on Math 9*, pp. 330, 331, 337, 342
- *TLE 9*, Evaluating Polynomials, Student Refresher pp. 44–45, Teacher’s Manual pp. 100–103

Previously Authorized Resources
- *Journeys in Math 8*, pp. 344–345
- *Journeys in Math 9*, pp. 128–129, 136, 143
- *Math Matters: Book 2*, pp. 87–88, 92–94, 112–113, 136–137

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Substituting values for the variables of an algebraic expression or formula is a useful skill. It is important that students learn and use proper substitution techniques, such as the use of parentheses and the order of operations. 1. Evaluate the expression, if $x = 3$ and $y = -1$. a. $x^2 - y^2$ b. $-3x + y$ c. $15 - xy$ d. $\frac{-x}{4y}$ e. x^y f. $\frac{x}{5} + \frac{y}{5}$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Solution

a. $x^2 - y^2 = (3)^2 - (^-1)^2$
$= 9 - 1$
$= 8$

b. $-3x + y = -3\ (3) + (^-1)$
$= -9 - 1$
$= -10$

c. $15 - xy = 15 - (3)\ (^-1)$
$= 15 - (^-3)$
$= 18$

d. $\frac{-x}{4y} = \frac{-(3)}{4(-1)}$
$= \frac{-3}{-4}$
$= \frac{3}{4}$

e. $x^y = (3)^{-1}$
$= \frac{1}{3}$

f. $\frac{x}{5} + \frac{y}{5} = \frac{3}{5} + \frac{-1}{5}$
$= \frac{2}{5}$

2. Complete the following:

x	$x^3 - 1$
2	
–3	
½	
6	
1	

Solution:

x	$x^3 - 1$
2	7
–3	–28
½	–⅞
6	215
1	0

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Use the formula $A = \frac{h(a+b)}{2}$ to calculate the area of a trapezoid with the following measurements: base a = 22 cm; base b = 15 cm; and height h = 6 cm. Solution $A = \frac{h(a+b)}{2}$ $A = \frac{6(22+15)}{2}$ $A = 111$ The area of the trapezoid is 111 cm^2.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

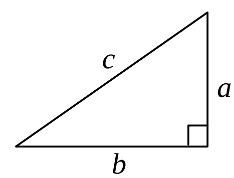
Paper and Pencil

1. Complete the following tables of values.

x	$3(x - 2)$
7	
–3	
1	
–2	

x	$2x(3 - x)$
1	
3	
–3	
4	

2. The formula for the length of the hypotenuse on the right triangle shown is c = $\sqrt{a^2 + b^2}$. Use proper substitution techniques to determine the length of the hypotenuse if the other two sides are 11.4 cm and 15.2 cm.



3. The formula for the surface area of a soup can is $A = 2\pi r\ (r + h)$, where r is the radius and h is the height of the can. Use proper substitution techniques to determine the surface area of a soup can that has a radius of 4.2 cm and a height of 10 cm.

4. Verify the following equations, by substituting 4 for x and –3 for y.

   a. $-2\ (x - y) = -2x + 2y$
   b. $2x\ (3x - 5y) = 6x^2 - 10xy$
   c. $(x - 3y)\ (x + 3y) = x^2 - 9y^2$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOMES

7. Represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams. [C, R, V] (9–9)
8. Perform the operations of addition and subtraction on polynomial expressions. [R] (9–10)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 72–75, 79–81
- *Interactions 9*, pp. 112–117
- *Mathpower 7*, pp. 230–231
- *Mathpower 8*, pp. 178–179
- *Mathpower 9*, pp. 72–73, 146, 150–155
- *Minds on Math 9*, pp. 327–331
- *TLE 9*, Algebra Tiles Explorer
- *TLE 9*, Adding and Subtracting Polynomials with Tiles, Student Refresher pp. 46–49, Teacher's Manual pp. 104–111

Previously Authorized Resources

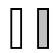
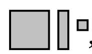
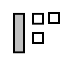
- *Journeys in Math 9*, pp. 128–131
- *Math Matters: Book 2*, pp. 89–91

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Magic number problems and calendar problems are both fun ways to practise the addition and subtraction of polynomials.
Refer to Portfolio Question 2 in the Tasks for Instruction and/or Assessment section.	Using manipulatives, such as algebra tiles, to introduce polynomials and their operations is useful for most students, particularly those who are still at the concrete stage of developing a conceptual understanding. When first using algebra tiles, give students time to become familiar with them. The coloured sides represent positive values, and the white sides represent negative values. Algebra tiles are particularly useful for operations with polynomials. Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

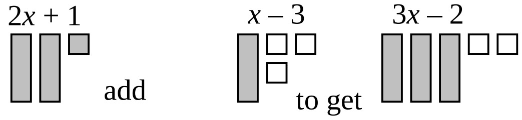
While addition of polynomials is often straightforward, consideration should be given to the different representations for subtraction, including the following:

- **comparison**—refers to comparing and finding the difference between two quantities
- **zero pairs**—think of two types of zero pairs
   : 0 = –1 + 1
   : 0 = $-x + x$
- **taking away**—simply refers to starting with a quantity and removing or taking away a specified amount to arrive at an answer. For example, for $(x^2 + 2x - 2) - (x^2 + x + 1)$, if we start with  and add a zero pair represented by  , then take away , the result is 
- **adding the opposite**—refers to subtracting by first changing the question to an addition and then adding the opposite of a quantity. For example, instead of subtracting x, one might add $-x$. Likewise, instead of subtracting $2x - 1$, one might add $-(2x - 1)$, which is the same as $-2x + 1$. Students should model $2x - 1$ and understand that the opposite is found by flipping the tiles.

  $2x - 1$ opposite $-2x + 1$

  
- **missing addend**—asks the question, "What would be added to the number being subtracted to get the starting amount?" For example, for $(3x - 2) - (2x + 1)$, ask: "What is added to $2x + 1$ to get $3x - 2$?"

  $2x + 1$ $x - 3$ $3x - 2$

Perimeter is a very useful application of addition and subtraction of polynomials.

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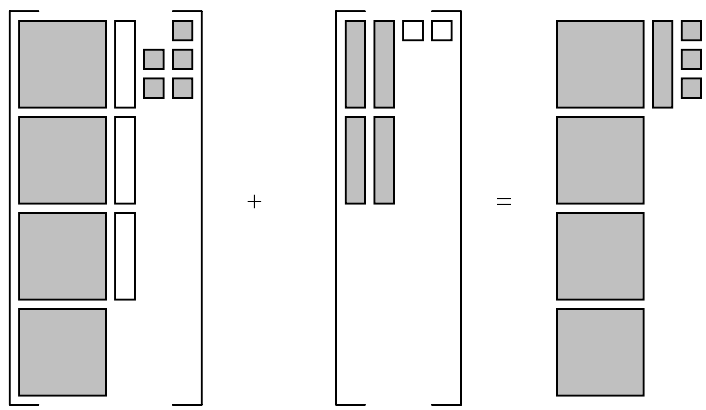
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	**Simplifying Expressions**

Simplifying Expressions

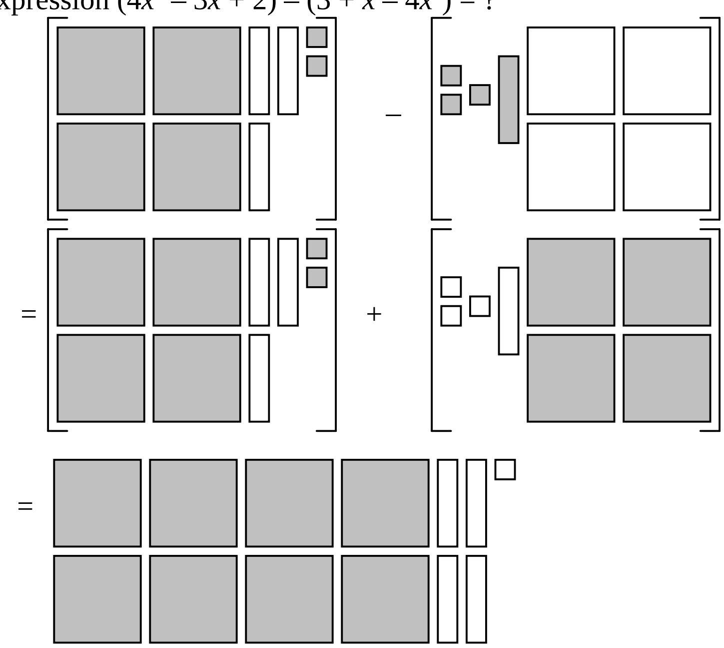
- **use algebra tiles to illustrate the combining of polynomials**

Examples

1. Demonstrate how the algebra tiles shown below are used to simplify the expression $(4x^2 - 3x + 5) + (4x - 2) = ?$



2. Explain how the algebra tiles shown below are used to simplify the expression $(4x^2 - 3x + 2) - (3 + x - 4x^2) = ?$



	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

3. Explain how algebra tiles can be used to illustrate the algebraic process for simplifying the following.
   a. $(2x^2 + 3x + 2) + (x^2 - 5x - 1)$
   b. $(4x^2 - 2x - 3) - (2x^2 - 3x)$

- **recognize and combine like terms**

Stress to students that the sign in front of any term must go with it.

Right	Wrong
$a - b + c$	$a - b + c$
$a + c - b$	$a - c + b$

Rearranging terms makes no difference to the result. This can be demonstrated easily with algebra tiles. However, discuss with students how reordering terms can aid in mental mathematics.

Examples

1. Which of these have the same value as 38 + 46 – 13 – 8 + 25?
   a. 38 – 13 + 46 + 25 – 8
   b. 38 + 25 + 46 – 13 – 8
   c. 38 – 46 + 13 – 8 + 25

2. Which of these expressions are equivalent to $a - b + c - d$?
   a. $a + c - b - d$
   b. $a - d - b + c$
   c. $a + c - d + b$
   d. $a + d - b + c$

3. The terms $9ab$ and $-9ab$ are like terms. What single change could you make to one of the terms so that they would be unlike terms? State at least three possible changes.

4. Combine like terms.
   a. $7x - 5x + x$
   b. $2b - 3 - 5b + 1$
   c. $-x - 3y + 6x + y$
   d. $-3xy + 5xz - 4xy - 4xz$

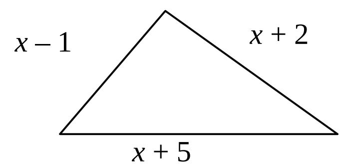
5. Simplify:
   a. $5x - 7x + 2x$
   b. $-5xy + 3xy$
   c. $(3x - 8) - (x^2 + 5x - 6)$
   d. $(5x^3 + 3m + 2) + (-2x^3 + 5p - 6)$
   e. Subtract $(-2x^2 + 5x - 3)$ from $(5x^2 - 3x + 7)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	6. The perimeter of a quadrilateral is $(10x + 4y + 16)$. What is a possible expression for the length of each side?

7. Simplify $3a - 8b + 7a - 15b + 10$.

8. C represents the number of compact discs, and $C + C + 4 + 2C = 56$. Using this information, write a problem.

9. Write an expression for the perimeter of the triangle below.



- **remove parentheses, and combine like terms**

Examples

1. $-(3x - y) = ?$
2. $7 - (p - 1) - (1 - p) = ?$
3. $(b - 3a) + (1 - 2b) - (2a + 5) = ?$
4. Subtract $(-2x + 2)$ from $(2x - 7)$.
5. $-(5 - 6x) - [-(6 - 5x - 2)] = ?$
6. $-2(2y + 1) - [-(2y - 1)] = ?$
7. One expression has been subtracted from another. What might the expressions be, if the difference is $-3x^2 - 4$?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Journal

1. Describe to a student who was absent for today's lesson how to use algebra tiles to add polynomials.

2. Describe how you feel about using algebra tiles to help you learn mathematics.

Performance

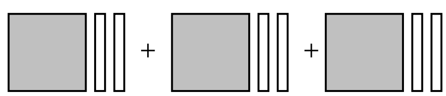
1. Using algebra tiles, illustrate two binomials whose sum is $-x + 2$. Give three solutions.

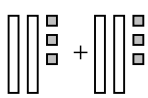
2. What binomial is missing? Tiles may be used.
   $(3x + 1) + (\qquad) = 2x + 3$

3. Ask students to show, through the use of algebra tiles, how the solutions to the following differ from each other.
   a. $(2x^2 + x) + (-4x^2 + 5x)$
   b. $(2x^2 + x) - (-4x^2 + 5x)$

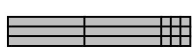
Paper and Pencil[1]

1. Ask students to record, symbolically, an expression for each of the following.

   a. 

   b. 

2. Ask students to write the dimensions and area for the rectangle shown.

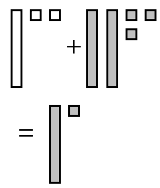


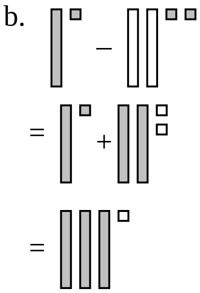
[1] Paper and Pencil questions 1 to 4 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*. Questions 11 to 13 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft*.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

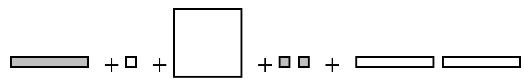
Teaching Notes

3. Write each of the steps in the problems that follow, using symbols, and explain each step.

a. 

b. 

4. Simplify the following expression.



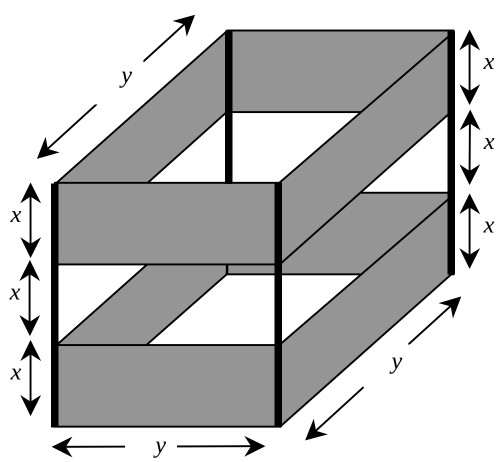
5. State three algebraic examples of two binomials whose difference is $2x – 3$.

6. Find k, if $–3x + 5x + kx = 7x$.

7. Find the value of k and t, if $3x + 2y – x + 4y = kx + ty$.

8. Box kites are made from lengths of wire, with fabric wrapped around them.

Write an expression for the length of wire needed for the kite shown below.



9. Write an algebraic expression for the quantity illustrated by the algebra tiles.

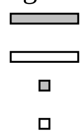
a.

b.

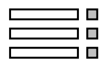
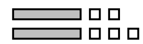
Strand: Patterns and Relations (Variables and Equations)
Specific Outcomes: 7. Represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams. [C, R, V] (9–9)
8. Perform the operations of addition and subtraction on polynomial expressions. [R] (9–10)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

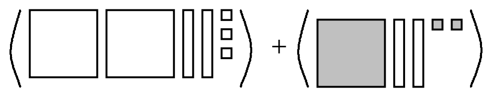
10. Write an algebraic expression for the quantity illustrated by the algebra tiles, if:

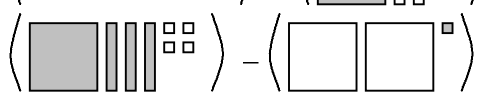


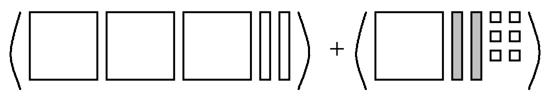
represents $+y$
represents $-y$
represents $+1$
represents -1

a.    b. 

11. Write an algebraic expression for each of the following, and simplify.

a. 

b. 

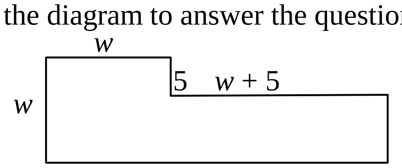
c. 

12. Sketch models to illustrate each algebraic expression, and use the models to simplify.
   a. $(2x^2 - 5x) - (-3x^2 + 2x)$
   b. $(-3y^2 - 2xy) + (y^2 + 4xy)$

13. List three different pairs of polynomials whose:
   a. sum is $3w^2 - 5w + 4$
   b. difference is $3w^2 - 5w + 4$.

Portfolio❶

1. Use the diagram to answer the questions below.



   a. Find a polynomial expression that represents the perimeter.
   b. If $w = 8$, find the perimeter using two forms of the polynomial expression. Which calculation was easier? Why?

❶ Portfolio questions 1 and 2 are reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. The following array represents a calendar for September.

	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Note that when any two-by-two array is selected from this calendar that the sum of the diagonal numbers is always the same. For example:

12 13   12 + 20 = 19 + 13
19 20

a. If we let x equal the first number in the two-by-two array, what will the number to the right and the number below it equal?
b. Express the sum of the diagonal numbers algebraically.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME 9. Represent multiplication, division and factoring of monomials, binomials and trinomials of the form $x^2 + bx + c$ using concrete materials and diagrams. [R, V] (9–11)

MANIPULATIVES

- Graph paper
- Algebra tiles
- Base-ten blocks

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 9*, pp. 118–129
- *Mathpower 9*, pp. 158–161, 188–203
- *Minds on Math 9*, pp. 338–345, 350–353, 358–365
- *TLE 9*, Binomial Grid Explorer
- *TLE 9*, Factoring Polynomials with Tiles, Student Refresher pp. 50–51, Teacher's Manual pp. 112–115

Previously Authorized Resources

- *Journeys in Math 9*, pp. 132–135, 138–147

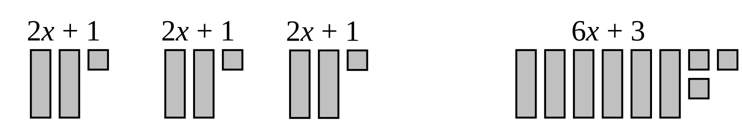
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	See below

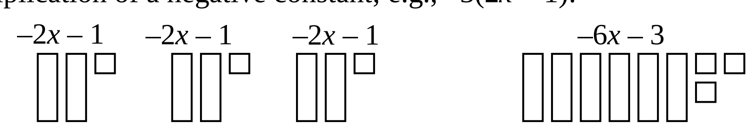
Multiplication of a Polynomial by a Constant

Algebra tiles can be used by students to do a variety of questions; e.g., 3(x + 2)—use three groups of x + 2, and simplify the result.

Multiplication of a polynomial by a constant should be developed with concrete materials and diagrams, using repeated addition. Given a problem such as 3(2x + 1), students should recognize that it is the same as 2x + 1 + 2x + 1 + 2x + 1 and, therefore, model the binomial three times, combine the like terms and arrive at an answer, as shown below.



Multiplication of a negative constant; e.g., –3(2x + 1):



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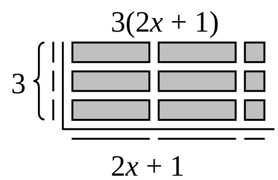
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	See below

The area model should also be explored in association with the topic, so that students can relate results achieved through repeated addition with results achieved using the area model.



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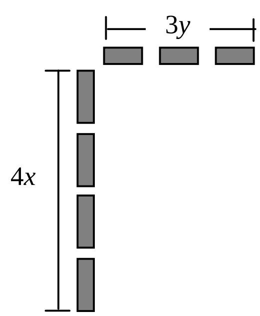
Multiplication of a Monomial by a Monomial

One method of illustrating multiplication of polynomials is through the use of area models. Base-ten blocks or algebra tiles are very helpful.

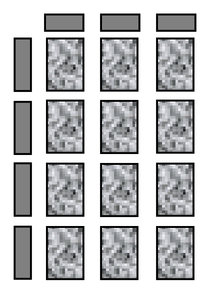
Example

1. Use algebra tiles and an area model to explain the multiplication $(4x)(3y)$.
   a. Set up the model by drawing a frame with dimensions $4x$ and $3y$.
   b. Show how to fill the area model in to get the product.

a.



b.



$12xy$

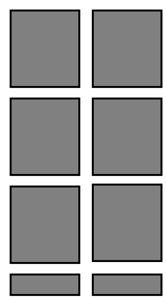
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Division of a Monomial by a Monomial

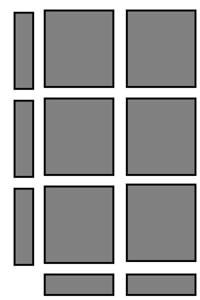
1. Use algebra tiles and an area model to explain the division $\dfrac{6x^2}{2x}$.
   a. Set up an area model, using six x^2 tiles, with $2x$ as one of the dimensions.
   b. Identify the other dimension by completing the frame. This will give the solution to the division question.

a.



($2x$)

b.

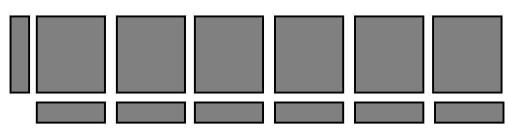


The solution to $\dfrac{6x^2}{2x}$ is $3x$.
Thus, the factors of $6x^2$ are ($3x$) and ($2x$).

Challenge: Find another set of factors for $6x^2$, using algebra tiles.

Solution:

The dimensions/factors could be $6x$ and x, as shown in the diagram below.



INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

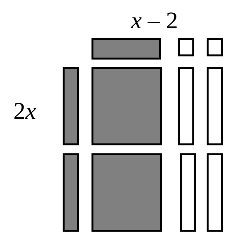
Multiplication and Division of a Polynomial by a Monomial

Multiplication of a Polynomial by a Monomial

Similarly, multiplication of a polynomial by a monomial can be demonstrated concretely with area models.

Example

1. Explain why the area model with algebra tiles can justify the product $2x(x – 2) = 2x^2 – 4x$.



Division of Monomials and Polynomials by Monomials

You determined the width of the rectangle by dividing the area by the known dimension. Similarly, you can divide a polynomial by a monomial that may or may not contain a variable.

Examples

1. Divide $6x^2$ by 3.

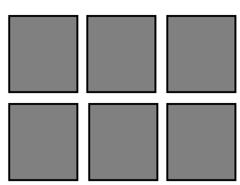
   Solution

   Divide the tiles into three equal groups. There are two x^2 tiles in each group. Therefore, $6x^2 \div 3 = 2x^2$.

2. Divide $6x^2$ by $3x$.

   Solution

   Use tiles to represent $6x^2$.

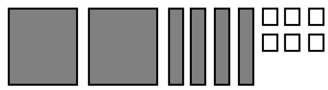


The width is $2x$,
so $\frac{6x^2}{3x} = 2x$.

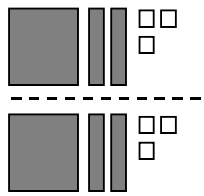
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Divide $(2x^2 + 4x - 6)$ by 2.

Solution

Use tiles to represent $2x^2 + 4x - 6$



Divide the tiles into two equal groups.



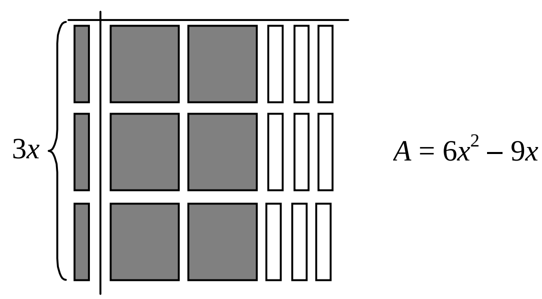
Count the number of tiles of each type there are in each group.

There is one x^2 tile, two x tiles and three negative unit tiles in each group. Therefore, $(2x^2 + 4x - 6) \div 2 = x^2 + 2x - 3$.

4. Divide $6x^2 - 9x$ by $3x$.

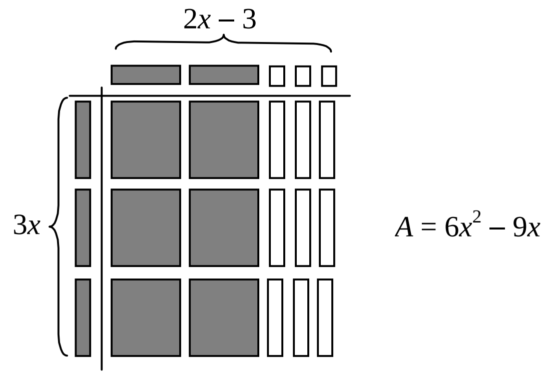
Solution

Use algebra tiles to create an area model of a rectangle with an area of $6x^2 - 9x$ and a width of $3x$.



INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Now use the area model to find the length of the rectangle.



The length of the rectangle is $2x - 3$.
Therefore, $(6x^2 - 9x) \div 3x = 2x - 3$.

Once several rectangles are constructed, students can look at the dimensions of a rectangle and relate the dimensions to the factors. They will then come to understand that the factors are the dimensions of a rectangle, whereas the area is the product of the factors. They can then be given the dimensions (factors) and asked to find the area (product). The product of $(x + 2)(x + 3)$ is the area of a rectangle that has a length of $x + 3$ and a width of $x + 2$. Students should understand that the x tile has an area of x square units but is also x units in length and 1 unit in width.

To model, the rectangle should be subdivided into four regions. Each region is filled with particular tiles according to the following scheme:

- region 1: x^2 tiles only
- region 2: horizontal x tiles
- region 3: vertical x tiles
- region 4: rectangular array of unit tiles

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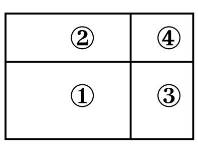
Multiplication of a Binomial by a Binomial

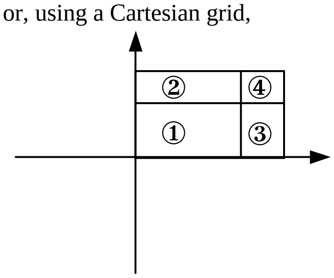
For multiplication of binomials, start by giving students a collection of algebra tiles, such as $x^2 + 6x + 5$, and ask them to create a rectangle by placing the unit tiles at the corner of the x^2 tiles and the x tiles along the sides of the x^2 tiles. When they create the rectangle they should spend some time reflecting on whether it is the only possible rectangle for the given materials. Students should record the dimensions of the rectangle. They should repeat this process several times with several trinomials until they are used to how to place the tiles. Then reverse the process, giving students the dimensions for a rectangle and asking

Teaching Notes

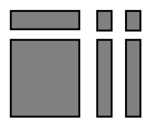
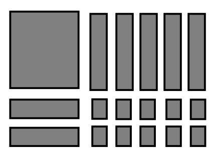
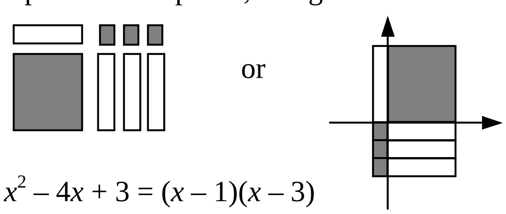
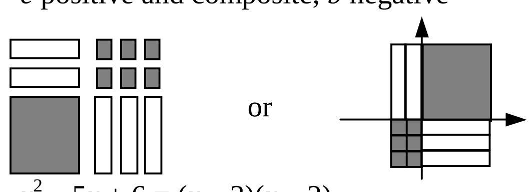
Either



or, using a Cartesian grid,



with region 4 at any corner of region 1, depending on the signs of b and c.

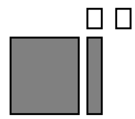
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	them to fill in the area and then record the value of that area. Since students should already know that $l \times w = A$, they can record their findings as a product and begin establishing the product and factors relationship. By comparing the symbols representing dimensions with those representing area, using several examples of both cases, and by recording their observations, students should establish a pattern. There should be a systematic order for introducing the trinomials. The following order is suggested. 1. *c* positive and prime, *b* positive  $x^2 + 3x + 2 = (x + 2)(x + 1)$ 2. *c* positive and composite, *b* positive  $x^2 + 7x + 10 = (x + 5)(x + 2)$ 3. *c* positive and prime, *b* negative  $x^2 - 4x + 3 = (x - 1)(x - 3)$ 4. *c* positive and composite, *b* negative  $x^2 - 5x + 6 = (x - 3)(x - 2)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

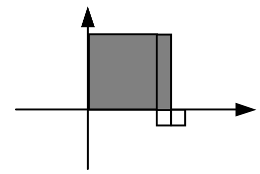
Teaching Notes

5. *c* negative and prime, *b* anything

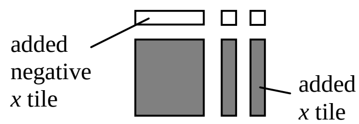
   Here zero pairs have to be added to complete the picture.
   $x^2 + x - 2$



or



   Now add one *x* tile and one negative *x* tile to complete the area.



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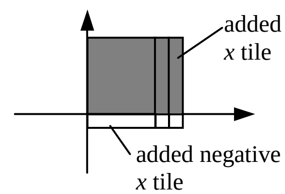
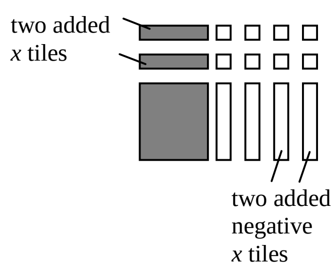
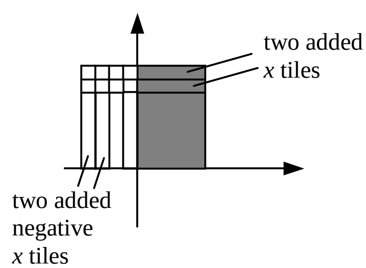


   $x^2 + x - 2 = (x + 2)(x - 1)$

6. *c* negative and composite, *b* anything
   $x^2 - 2x - 8$



or



   $x^2 - 2x - 8 = (x - 4)(x + 2)$

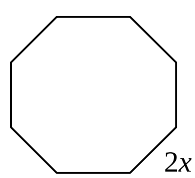
Tables can also be used for multiplication of polynomials;
e.g.: $(a + 3)(a - 9)$

	a	$+3$
a	a^2	$3a$
-9	$-9a$	-27

$(a + 3)(a - 9) = a^2 - 6a - 27$

Sample Questions

1. Express the perimeter of the regular octagon in terms of *x*.

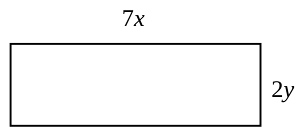


	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

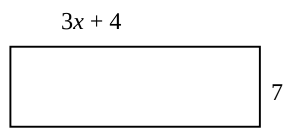
Teaching Notes

2. Use a rectangular area model:

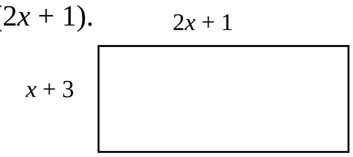
   a. with dimensions $7x$ and $2y$ to find the product $(7x)(2y)$



   b. with dimensions $3x + 4$ and 7 to find the product $(7)(3x + 4)$



   c. with dimensions $(x + 3)$ and $(2x + 1)$ to find the product $(x + 3)(2x + 1)$.

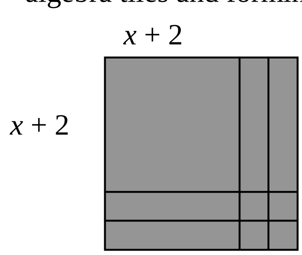


3. Use algebra tiles to represent each of the following products.
   a. $2x(x + 3)$
   b. $3(2x + 1)$
   c. $(x + 2)(2x + 1)$
   d. $(2x + 1)(x – 1)$

4. Use algebra tiles to represent the factoring of the following polynomials.
   a. $6x + 9$
   b. $2x^2 + 6x$
   c. $x^2 + 8x + 12$
   d. $x^2 + 4x + 3$

5. Use algebra tiles to represent the following division.
   $(6x^2 + 8x) \div (2x)$

6. Natalia modelled the process of factoring $x^2 + 4x + 4$, by using algebra tiles and forming a square with them.



What are the factors of $x^2 + 4x + 4$?

Use Natalia's method to factor $x^2 + 5x + 6$.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	7. Ask students to write, symbolically, the dimensions and area for the rectangle shown.  8. Ask students to present the product for each of the following, using algebra tiles or diagrams. a. $2(x^2 + 3)$ b. $3(2x - 1)$ c. $3(x^2 - 2x + 1)$ 9. Ask students to show the product of 3 and $2x + 4$ as the area of a rectangle.

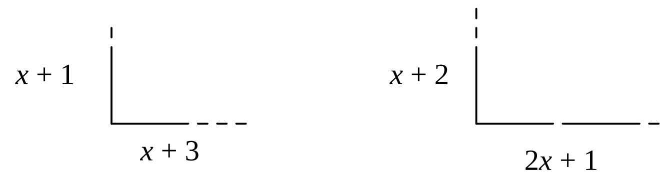
Strand: Patterns and Relations (Variables and Equations)
Specific Outcome: 9. Represent multiplication, division and factoring of monomials, binomials and trinomials of the form $x^2 + bx + c$ using concrete materials and diagrams. [R, V] (9–11)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	See tasks below

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Performance

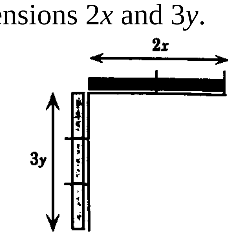
1. Create the product rectangle, using algebra tiles, and record the factors and the product symbolically.



2. In the first diagram, create a rectangle for the dimensions shown; and in the second, record the dimensions for the given area. Compare and discuss the results.

3. Represent the binomial $12x – 6$ with area models, using algebra tiles, in three different ways. Using the models, write $12x – 6$ as the product of a constant and a binomial.

4. a. Justin used algebra tiles and an area model to explain the multiplication $(2x)(3y)$. He set up the model by drawing a frame with dimensions $2x$ and $3y$.



   Show how he filled in the area model to get the product.

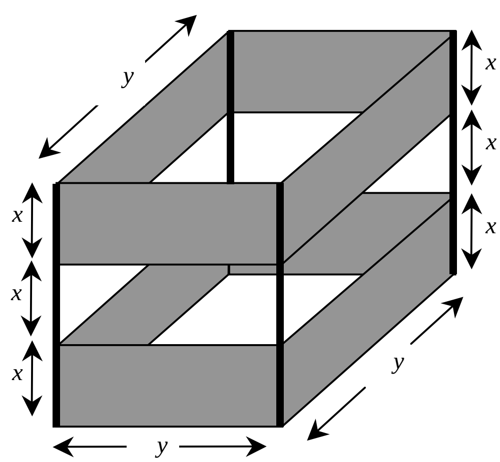
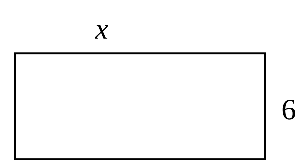
   b. Use an area model with algebra tiles to find and justify the product $2(x – 2)$.

5. Form $x^2 + 5x$, using algebra tiles.
   a. Create a rectangle where x is one of the dimensions.
   b. What is the other dimension?
   c. Write a division sentence for the situation.

Portfolio/Journal❶

1. Show how the process of multiplying 3 × 2 is similar to the process of multiplying $(x + 1)(x + 3)$, by using area models.

❶ Portfolio/Journal questions 2 and 3 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft*.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. Examine the pattern of the factors for each of the following: $x^2 + 2x + 1$, $x^2 + 4x + 4$, $x^2 + 6x + 9$, $x^2 + 8x + 16$. a. Explain the pattern that you observe. b. Create at least two other polynomials whose factors are consistent with this pattern. c. Can you think of a concise way of writing the factors? d. Use what you have learned in parts a–c to find each of the following: $(x + 5)^2$, $(x + 6)^2$, $(x + a)^2$, $(x + 2b)^2$. 3. Examine the pattern of the factors for each of the following: $x^2 + 3x + 2$, $x^2 + 4x + 3$, $x^2 + 5x + 4$. a. Explain the pattern that you observe. b. Create at least two other polynomials whose factors will be consistent with this pattern. c. Use the pattern you have observed to find each of the following products quickly: $(x + 1)(x + 5)$, $(x + 1)(x + 9)$, $(x + 1)(x + 50)$. 4. Box kites are made from lengths of wire, with fabric wrapped around them. Using the diagram of the box kite, write an expression for the amount of fabric used.  5. The width of a rectangle is 6 and the length is x. If the length of the rectangle is increased by 3, by how much does the area increase? 

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME 10. Find the product of:

- two monomials
- a monomial and a polynomial
- two binomials.

[R] (9–12)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 76, 78, 84–91
- *Interactions 9*, pp. 118–120, 150–152, 159–161
- *Mathpower 9*, pp. 158–161, 188–189, 193–194, 198–199, 202–203
- *Minds on Math 9*, pp. 334–342, 350–355
- *TLE 9*, Multiplying Polynomials, Student Refresher pp. 52–53, Teacher's Manual pp. 116–119

Previously Authorized Resources

- *Journeys in Math 9*, pp. 132–135, 142–143
- *Math Matters: Book 2*, pp. 106–113, 118–128

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Before teaching multiplication of polynomials, it may be useful to review the following concepts with students: • polynomial vocabulary—coefficient, variable, monomial, binomial • multiplication of integers • addition of integers • exponent laws • distributive property. To multiply two monomials, multiply their coefficients and multiply their variables. $2x^2y(-4xy) = -8x^3y^2$ To multiply a monomial and a polynomial, use the distributive property. $2x(5x^2 - 3x + 1) = 10x^3 - 6x^2 + 2x$

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

To multiply two binomials, use the distributive property twice, then simplify.

$$(x - 3)(2x + 1) = 2x^2 + x - 6x - 3$$
$$= 2x^2 - 5x - 3$$

1. Multiply.
   a. $(-3x)(4xy)$
   b. $(5a^2b)(2a^3b)$
   c. $(-7m)(4)$
   d. $(-xy)(-x^3y^4)$
   e. $(8d^3ef)(-ef^2g)$

   Solution

   a. $-12x^2y$
   b. $10a^5b^2$
   c. $-28m$
   d. x^4y^5
   e. $-8d^3e^2f^3g$

2. Multiply and simplify.
   a. $2x(5 - 3x + x^2)$
   b. $-5y(-y - 8)$
   c. $(4xy^3 - x^2 + 9)8y$
   d. $7(-10p - q^3 - 6)$
   e. $-2x(x + 5) + 3x(x - 1)$
   f. $4x(y + 2) - (y - 1)$

   Solution

   a. $10x - 6x^2 + 2x^3$
   b. $5y^2 + 40y$
   c. $32xy^4 - 8x^2y + 72y$
   d. $-70p - 7q^3 - 42$
   e. $-2x^2 - 10x + 3x^2 - 3x = x^2 - 13x$
   f. $4xy + 8x - y + 1$

3. Expand and simplify.
   a. $(x + 5)(x - 4)$
   b. $(x - 3)(x - 2)$
   c. $(x + 6)(x + 4)$
   d. $(-2x + y)(3x - 4)$
   e. $(3 - 2m)(4 + 6m)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Solution a. $x^2 – 4x + 5x – 20 = x^2 + x – 20$ b. $x^2 – 2x – 3x + 6 = x^2 – 5x + 6$ c. $x^2 + 4x + 6x + 24 = x^2 + 10x + 24$ d. $–6x^2 + 8x + 3xy – 4y$ e. $12 + 18m – 8m – 12m^2 = 12 + 10m – 12m^2$

Strand: Patterns and Relations (Variables and Equations)
Specific Outcome: 10. Find the product of: two monomials, a monomial and a polynomial, two binomials. [R] (9–12)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

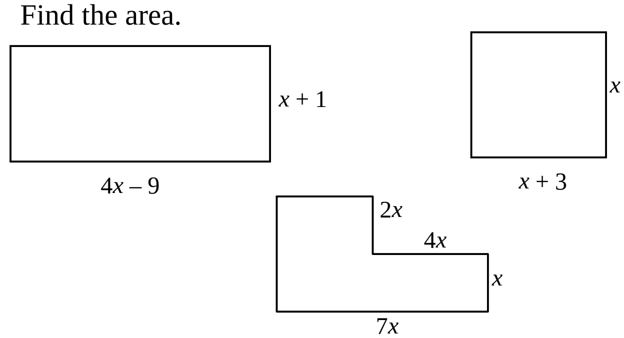
Teaching Notes

Paper and Pencil

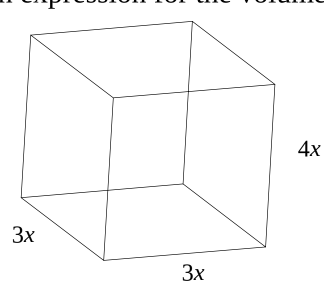
1. Complete the table.

Factor	Factor	Product
(x^3)	$2x + 9$	
$-2ab$	$-6a$	
$x - 1$	$2x + 3$	
$-5a^2bc$	$2abc$	
$-4x$	$x^3 - x^2 + 2$	
$2\ell^2m^2n^2$	$3\ell^2mn^4$	
$(x - 6)$	$(x - 4)$	
$8 - m$	$7 + m$	

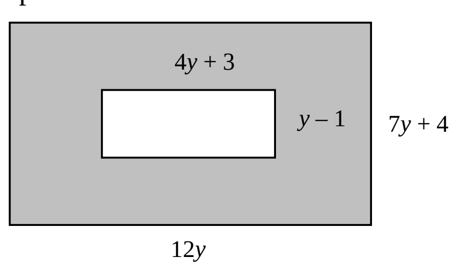
2. Find the area.



3. Write an expression for the volume. Find the volume.



4. Write an expression for the shaded area.



STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

11. Determine equivalent forms of algebraic expressions by identifying common factors. [PS, R] (9–13)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 93–96
- *Interactions 9*, pp. 154–158
- *Mathpower 9*, pp. 178, 182–186
- *Minds on Math 9*, pp. 343–347
- *TLE 9*, Dividing Polynomials by Monomials, Student Refresher pp. 54–55, Teacher's Manual pp. 120–123

Previously Authorized Resources

- *Journeys in Math 9*, pp. 144–145
- *Math Matters: Book 2*, pp. 121–123, 134

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Before teaching common factoring, it may be useful to review the following concepts with students: • greatest common factors of numbers • prime factorization of numbers • divisibility rules • division of monomials by monomials. Students should understand that factoring is the reverse of multiplication—what multiplication does, factoring undoes. Multiplication: $2 \times 3 = 6$; $(5x)(2x) = 10x^2$; $2(x + 3) = 2x + 6$ Factoring: $6 = 2 \times 3$; $10x^2 = (5x)(2x)$; $2x + 6 = 2(x + 3)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

It may be advisable to begin with something students have done before—the prime factorization method of determining the greatest common factor (GCF) of whole numbers.

$$\left.\begin{aligned}15 &= 3\bullet 5\\ 9 &= 3\bullet 3\end{aligned}\right\}\ \text{GCF} = 3$$

$$\left.\begin{aligned}18 &= 3\bullet 3\bullet 2\\ 12 &= 3\bullet 2\bullet 2\end{aligned}\right\}\ \text{GCF} = 3\bullet 2 = 6$$

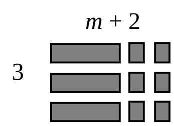
Extend the above concept to monomials.

$$\left.\begin{aligned}3m &= 3\bullet m\\ 6 &= 3\bullet 2\end{aligned}\right\}\ \text{GCF} = 3$$

$$\left.\begin{aligned}m^3 &= m\bullet m\bullet m\\ 2m^2 &= 2\bullet m\bullet m\end{aligned}\right\}\ \text{GCF} = m\bullet m = m^2$$

Then show how this applies to polynomials.

$3m + 6 = 3(m + 2)$



TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

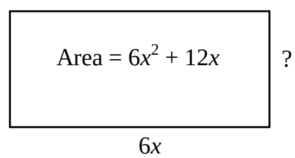
Paper and Pencil

1. Find the factors.
   a. $10x + 5$
   b. $2x^2 + 14x$
   c. $12x - 18y$
   d. $9c^3d + 5c^2 - 7c^2d$
   e. $5b^2c^2d^2 + 15bc - 10bcd^2$

2. The area of a rectangle is $10x^3y^2 + 5xy^2 - 15x^2y$ and the width is $5xy$. Find the length and the perimeter.

3. Use algebra tiles to factor $4x^2 - 8x$.

4. Find the missing expression for the width.



5. Complete the table.

Polynomial	GCF	Other Factor
$27r^3 - 18r^2 + 9r$	$9r$	
$14abc - 21ab^2$	$7ab$	
	$8y^2$	$3xz - 2y - 1x$
$2x^2y^2 + 8xy$		$xy + 4$
$6y^2 - 3xy + 9x^2y$		$2y - x + 3x^2$
	$4a$	$2a - b + 6$
$10x - 5y + 15$	5	

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

12. Factor trinomials of the form $ax^2 + bx + c$ where $a = 1$, or of the form $ax^2 + abx + ac$. [PS, R] (9–13)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 98–100
- *Interactions 9*, pp. 162–165
- *Mathpower 9*, pp. 195–197
- *Minds on Math 9*, pp. 358–365
- *TLE 9*, Factoring Polynomials, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources

- *Journeys in Math 9*, pp. 146–148
- *Math Matters: Book 2*, pp. 129–131, 134

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Since factoring is the reverse of multiplication, it may be helpful to review multiplication of simple binomials. Multiplication: $(x + 3)(x + 2) = x^2 + 5x + 6$ Factoring: $x^2 + 5x + 6 = (x + 3)(x + 2)$ The following steps could be used to factor a simple trinomial. 1. Split the x^2 term into its factors, and write them in the parentheses. $x^2 + 3x - 10 = (x \quad)(x \quad)$ 2. Determine all the factor pairs of the constant term. 10 and –1 –10 and 1 2 and –5 –2 and 5

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Determine which pair has a sum equal to the coefficient of the middle term, and write them in the parentheses. $x^2 + 3x - 10 = (x - 2)(x + 5)$ 4. Check the answer by multiplying the factors. If the trinomial to be factored has a common factor in each of its terms, the common factor should be removed first. $2x^2 + 10x + 12 = 2(x^2 + 5x + 6)$ $= 2(x + 2)(x + 3)$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

Paper and Pencil

1. Factor

   a. $x^2 + 10x + 25$
   b. $x^2 - x - 12$
   c. $x^2 + x - 20$
   d. $x^2 + 8x + 16$
   e. $x^2 - 5x + 4$

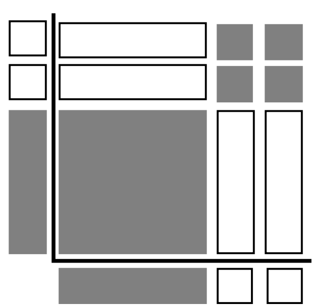
2. Remove the GCF and factor fully.

   a. $3x^2 + 6x - 9$
   b. $10x^2 - 60x + 80$
   c. $7x^2 + 42x + 56$
   d. $2x^2 - 4x - 30$
   e. $4x^2 - 8x - 60$

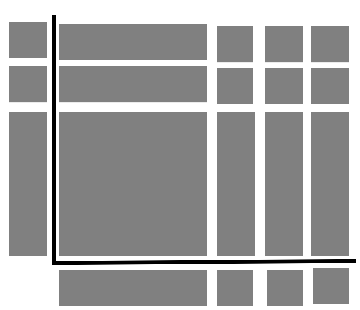
3. Identify the factors and the product for the diagrams below.
   (**Note**: Shaded is positive, white is negative)

   a.

   b.



   c.



	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

4. State the dimensions and perimeter for each diagram.

   a. Area: $x^2 + 7x + 10$

   b. Area: $x^2 - 11x + 18$

   c. Area: $x^2 + 4x - 21$

5. Find all of the possible solutions for □, assuming that the polynomial can be factored.

   a. $x^2 + \square x - 12$
   b. $x^2 - \square x + 14$
   c. $x^2 + \square x + 16$
   d. $x^2 - \square x - 20$

6. Complete the table.

Product	Factor	Factor
a. $x^2 - 3x - 18$	$(x + 3)$	
b.	$(x - 2)$	$(x + 5)$
c. $2x^2 + 22x + 20$	$2(x + 1)$	( )
d. $x^2 - 5x + 6$	$(x - 3)$	( )
e.	$3(x - 2)$	$(x + 8)$
f.	$(x + 1)$	$(x + 4)$
g.	$(2x + 3)$	$(2x + 3)$
h. $x^2 + 8x + 16$		

7. Change one term in the following trinomials to make the trinomial a perfect square.

   a. $x^2 - 25x + 100$
   b. $x^2 + 14x + 54$
   c. $x^2 - 16x - 64$
   d. $2x^2 + 18 + 81$

8. Identify the polynomials below that are perfect squares, and factor these polynomials.

   a. $x^2 - 22x + 121$
   b. $x^2 + 20x + 100$
   c. $x^2 + 2x + 4$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME 13. Factor polynomials of the form $A^2 - B^2$ where A and B are both monomial expressions. [PS, R] (9–11, 9–13)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 104–108
- *Interactions 9*, p. 163
- *Mathpower 9*, pp. 200–201
- *TLE 9*, Factoring Polynomials, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources

- *Math Matters: Book 2*, pp. 132–134

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	When faced with a factoring problem, it is important that students are able to recognize which type of factoring is involved—common, trinomial or difference of squares. When introducing difference of squares factoring, such vocabulary words as "square numbers" and "square roots" must be understood. Again, the concept of reverse operations can help. Multiplication: $(x + 2)(x – 2) = x^2 – 4$; $(x + 5)(x – 5) = x^2 – 25$ Factoring: $x^2 – 4 = (x + 2)(x – 2)$; $x^2 – 25 = (x + 5)(x – 5)$ The meaning of the phrase "difference of squares" should be discussed. It is also useful to discuss the reason that the middle term is missing in the product of $(x + 5)(x – 5) = x^2 – 25$. Once students understand these two concepts, most will be able to factor difference of squares binomials with little difficulty. Some may need the following steps.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	To factor a difference of squares: 1. Write the square root of the first term at the beginning of each parenthetical set. $x^2 - 16 = (x \quad)(x \quad)$ 2. Write the square root of the last term at the end of each parenthetical set. Include a plus sign in one and a minus sign in the other. $x^2 - 16 = (x + 4)(x - 4)$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. Factor a. $x^2 - 100$ b. $25x^2 - 49y^2$ c. $4a^2 - 9$ d. $16 - z^2$ 2. Remove the common factor, and factor fully. a. $81a^2b - 4b$ b. $32x^2 - 18y^2$ c. $27 - 12f^2$ d. $10y^2 - 10x^2$ e. $50 - 8x^2$ f. $36g^2 - 64$ **Performance** 1. Use algebra tiles to multiply $(x + 3)(x - 3)$, $(x + 2)(x - 2)$ and $(x + 5)(x - 5)$. Simplify each product. Describe the similarities in the three products. **Journal/Interview** 1. Why do you think the product of $(x - 6)(x + 6)$ is called a difference of squares? 2. Which of the following products are a difference of squares? Justify your answer. a. $x^2 - 121$ b. $125 + 100x^2$ c. $25x^2 - 16y$ d. $8x^2 - 50$ 3. Explain how to determine if a binomial product is a difference of squares.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

14. Find the quotient when a polynomial is divided by a monomial. [PS, R] (9–14)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 76–78
- *Interactions 9*, pp. 122–127
- *Mathpower 9*, pp. 164–165, 175, 178, 190–191
- *Minds on Math 9*, pp. 292–293, 334–337, 348–349
- *TLE 9*, Factoring Polynomials, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources
- *Journeys in Math 9*, pp. 138–141
- *Math Matters: Book 2*, pp. 114–117, 120

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Students should be exposed to the evaluation of polynomials (Patterns and Relations Specific Outcome 6) before and after they have been simplified so that the advantage of simplifying prior to evaluating can be emphasized. It is through such comparisons that students can see a need for simplifying.	There are two methods for division of a polynomial by a monomial. One method is to break the polynomial apart and solve individual monomial division problems; e.g., $\frac{3x+12}{3}=\frac{3x}{3}+\frac{12}{3}=x+4$. ❶ The second method is to factor the polynomial and simplify. $\frac{3x+12}{3}=\frac{\not{3}(x+4)}{\not{3}}=x+4$ 1. Divide $(3m^3+8m^2-5m)$ by $4m$. Solution—Method 1 $\frac{3m^3+8m^2-5m}{4m}=\frac{3m^3}{4m}+\frac{8m^2}{4m}+\frac{-5m}{4m}$ $=\frac{3m^2}{4}+2m+\frac{-5}{4}$ $=\frac{3}{4}m^2+2m+\frac{-5}{4}$ $=\frac{3}{4}m^2+2m-\frac{5}{4}$

❶ Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Solution—Method 2

$$\frac{3m^3 + 8m^2 - 5m}{4m}$$

$$= \frac{\not{m}(3m^2 + 8m - 5)}{4\not{m}}$$

$$= \frac{3m^2 + 8m - 5}{4} \quad \text{or} \quad \frac{3}{4}m^2 + 2m - \frac{5}{4}$$

Verification

Verify by substituting $m = 10$.

LS	RS
$\frac{3(10)^3 + 8(10)^2 - 5(10)}{4(10)}$	$\frac{3}{4}(10)^2 + 2(10) - \frac{5}{4}$
$= \frac{3000 + 800 - 50}{40}$	$= \frac{3}{4}(100) + 2(10) - \frac{5}{4}$
$= \frac{3750}{40}$	$= 75 + 20 - \frac{5}{4}$
$= \frac{375}{4}$ or $93\frac{3}{4}$	$= 93\frac{3}{4}$

LS = RS

2. Divide $(8x^3 + 4x^2 - 4x)$ by $-4x$.

Solution—Method 1

$$\frac{8x^3 + 4x^2 - 4x}{-4x} = \frac{8x^3}{-4x} + \frac{4x^2}{-4x} + \frac{-4x}{-4x}$$
$$= -2x^2 - x + 1$$

Solution—Method 2

$$\frac{8x^3 + 4x^2 - 4x}{-4x}$$

$$= \frac{\not{4}x(2x^2 + x - 1)}{-\not{4}x}$$

$$= -2x^2 - x + 1$$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Verification Verify by substituting $x = 10$. LS: $\frac{8(10)^3 + 4(10)^2 - 4(10)}{-4(10)}$ $= \frac{8000 + 400 - 40}{-40}$ $= \frac{8360}{-40}$ $= -209$ RS: $-2(10)^2 - (10) + 1$ $= -200 - 10 + 1$ $= -209$ LS = RS 3. Division is the same as multiplying by the reciprocal. $$\frac{a+b}{c} = \frac{1}{c}(a+b)$$ By applying the distributive property, the problem is reduced to one of dividing monomials. $$\frac{1}{c}(a+b) = \frac{1}{c} \bullet a + \frac{1}{c} \bullet b = \frac{a}{c} + \frac{b}{c}$$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

Paper and Pencil

1. a. Find the quotient of $\dfrac{12x^3 - 16x^2 + 8x}{4x}$ by dividing each term in the numerator by the term in the denominator.
   b. Find the quotient by factoring the numerator first. What do you notice?

2. Factor the numerator, then divide.
   a. $\dfrac{6x^3 + 4x^2 + 2x}{2x}$
   b. $\dfrac{4y^5 + 8y^3 - 2y^2}{2y^2}$

3. Perform the following divisions. Verify your answers by substituting $x = 10$.
   a. $(4x^3 + 2x^2 – 6x) \div 2x$
   b. $(6x – 12) \div (–3)$
   c. $\dfrac{4x^2 + 6x - 8}{5}$
   d. $\dfrac{8x^3 + 12x^2 - 4x}{-4x}$

4. Perform the following divisions.
   a. $(6x^3 + 4x^2 + 8x) \div 3x$
   b. $(12x^4 + 8x^3 – 16x^2 + 4x) \div (–4x)$
   c. $(x^3 – 9x) \div x$
   d. $\dfrac{-12y^3 + 9y^2 - y}{-3y}$
   e. $\dfrac{24 - 8m + 4m^2}{4}$
   f. $\dfrac{14a^2b^3 + 7a^2b^2 - 7ab^2}{7ab}$

5. a. Evaluate the expression $(4y^3 – 12y^2 + 8y) \div 4y$ for $y = –3$.
   b. Perform the division first, and then evaluate the expression once again for $y = –3$.
   c. Compare the two results.
   d. Was it simpler to evaluate the expression before or after division? Explain.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

6. Simplify the expression:

$$\frac{2x^2 + 6x}{2x}$$

   a. Find the value of the expression for $x = 6$, by replacing x in the original expression.
   b. Find the value of the expression for $x = 6$, by replacing x in the simplified expression.
   c. Compare the two results. What do you notice?

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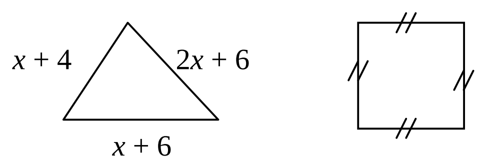
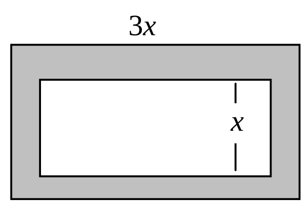
7. The perimeter of the triangle is equal to the perimeter of the square. Find the length of one side of the square in terms of x.



8. The inside rectangle in the diagram below is a flower garden. The shaded area is a concrete walk-way around it. The area of the flower garden is given by the expression $2x^2 – 4x$, and the area of the walk-way is $x^2 + 19x$.



   a. What is the total area of the flower garden and the concrete walk-way?
   b. Use the information provided to find an expression for each of the missing dimensions of each rectangle.
   c. If $x = 2.3$ m, find the dimensions and area of the flower garden.

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STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOME

15. Illustrate the solution process for a one-step, single variable, first-degree equation, using concrete materials or diagrams.
    - $x + a = b$
    - $x - a = b$
    - $ax = b$
    - $\frac{x}{a} = b$

    [CN, PS, V] (7–7)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 223–227
- *Interactions 8*, pp. 296–297
- *Mathpower 7*, pp. 228–235
- *Mathpower 8*, pp. 172, 180–183
- *Mathpower 9*, pp. 72–83
- *Minds on Math 7*, pp. 331–334, 338–341
- *Minds on Math 8*, pp. 371–376
- *Minds on Math 9*, pp. 133–138
- *TLE 8*, Linear Equations (1 Step Solution), Student Refresher pp. 44–45, Teacher's Manual pp. 100–103

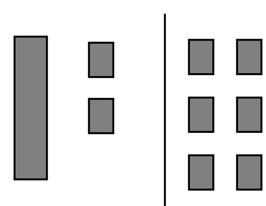
Previously Authorized Resources

- *Journeys in Math 8*, pp. 354–357
- *Journeys in Math 9*, pp. 164–165

TECHNOLOGY CONNECTIONS

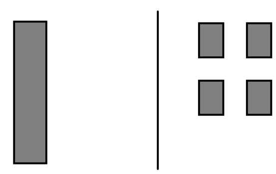
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Use algebra tiles to pictorially show one-step equations.

1. $x + 2 = 6$



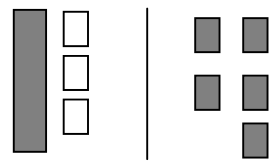
(take away 2 from each side)

$x = 4$

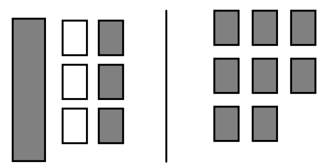


	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

2. $x - 3 = 5$

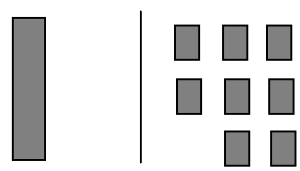


(add 3 to each side)

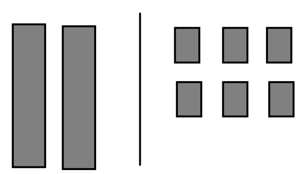


(combining, 3 sets of cancel out)

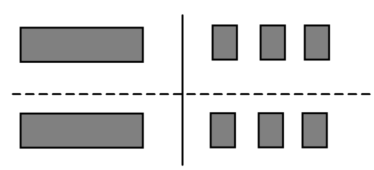
$x = 8$



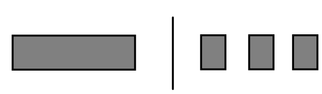
3. $2x = 6$



(regroup for 1)



$x = 3$



	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal/Interview** 1. Have students describe how they would set up specific equations with algebra tiles. 2. Have students describe the manipulation of tiles to solve these equations. **Paper and Pencil** 1. Use algebra tiles or diagrams to demonstrate the solution of the following. a. $a + 5 = 2$ b. $b - 3 = 6$ c. $c + 2 = -3$ d. $5d = -15$ e. $-3e = -12$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOMES

16. Solve and verify one-step linear equations of the form:
    - $x + a = b$
    - $\frac{x}{a} = b$
    - $ax = b$

    where *a*, *b* and *c* are integers, using a variety of techniques.
    [PS, R] (7–8)
18. Solve and verify one- and two-step first degree equations of the form:
    - $\frac{x}{a} + b = c$
    - $ax + b = c$

    where *a, b* and *c* are integers.
    [PS, V] (8–5)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 219–231
- *Interactions 8*, pp. 291–305
- *Interactions 9*, pp. 226–231
- *Mathpower 7*, pp. 200–201, 228–235, 244
- *Mathpower 8*, pp. 176–183, 188–193, 196, 198
- *Mathpower 9*, pp. 70–83, 88–91
- *Minds on Math 7*, pp. 344–363
- *Minds on Math 8*, pp. 372–385
- *Minds on Math 9*, pp. 133–143
- *TLE 8*, Linear Equations (1 Step Solution), Student Refresher pp. 44–45, Teacher's Manual pp. 100–103
- *TLE 8*, Solving Problems, Student Refresher pp. 48–49, Teacher's Manual pp. 108–111

Previously Authorized Resources
- *Journeys in Math 8*, pp. 348–359
- *Journeys in Math 9*, pp. 162–167

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS[1]
Teaching Notes	Students need to understand three essential concepts when solving 2-step, single-variable, first-degree equations. • The objective is to finish with *x* equal to a value. • The equation must always balance. • To move a number, undo it by using the inverse operation; that is, add the opposite quantity to a term or multiply by the inverse of the coefficient.

[1] The Instructional Strategies/Suggestions for these specific outcomes are reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	When students are familiar with solving equations using concrete materials, ask them to solve equations using formal methods, such as the following:

1. $x + a = b$, where a is positive or negative

$$\begin{aligned} x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x + 0 &= 10 \\ x &= 10 \end{aligned}$$

OR

$$\begin{aligned} x - 3 &= 7 \\ +3 &= +3 \\ \hline x + 0 &= 10 \\ x &= 10 \end{aligned}$$

2. $ax = b$

$$\begin{aligned} 4x &= 20 \\ \frac{4x}{4} &= \frac{20}{4} \\ x &= \frac{20}{4} \\ x &= 5 \end{aligned}$$

3. $\frac{x}{a} = b$, where $a \neq 0$

$$\begin{aligned} \frac{x}{3} &= 4 \\ \frac{x}{3} \times 3 &= 4 \times 3 \\ x &= 4 \times 3 \\ x &= 12 \end{aligned}$$

4. $ax + b = c$

$$\begin{aligned} 4x + 7 &= 19 \\ -7 &= -7 \\ \hline 4x + 0 &= 12 \\ 4x &= 12 \\ \frac{4x}{4} &= \frac{12}{4} \\ x &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

5. $\frac{x}{a} + b = c$, where $a \neq 0$

$$\begin{aligned} \frac{x}{3} + 5 &= 20 \\ -5 &= -5 \\ \hline \frac{x}{3} + 0 &= 15 \\ \frac{x}{3} \times 3 &= 15 \times 3 \\ x &= 15 \times 3 \\ x &= 45 \end{aligned}$$

Note: At first, ask students to use concrete materials to verify their solutions. Later, have them verify by substituting the value into the equation.

Verify $\frac{x}{3} + 5 = 20$, where $x = 45$

$$\begin{aligned} \frac{45}{3} + 5 &= 20 \\ 15 + 5 &= 20 \\ 20 &= 20 \end{aligned}$$

Left Side = Right Side

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	(see below)

Journal/Interview

1. What does it mean to verify a solution?
2. Describe how to verify a solution.

Performance

1. Provide students with a variety of 1- and 2-step equations to solve using concrete materials and have them record each step symbolically.

   <u>Observations</u>

   Check for the following:

   Does the student:
   - change the concrete form to its symbolic representation and vice versa?
   - solve 1-step, single-variable equations involving addition, subtraction, division or multiplication?
   - solve 2-step, single-variable equations involving:
     – addition and division?
     – subtraction and division?
     – addition and multiplication?
     – subtraction and multiplication?
   - solve the equation, given a value for the variable (solve for *y*, given *x*)?
   - determine the value of the variable, given the solution for the equation (solve for *x*, given *y*)?

   Observe and record student performance level:
   - independently with ease
   - independently with difficulty
   - only with assistance.

   Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

Paper and Pencil

1. Solve and verify the following.

   a. $x + 9 = -5$
   b. $x - 3 = -7$
   c. $-2x = 18$
   d. $\frac{x}{0.4} = -1.3$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. Solve and verify the following. a. $2x + 6 = 22$ b. $7 - 5x = -27$ c. $\frac{x}{-2} - 4 = 9$ 3. Jacques found the solution for $2x + 7 = 13$ to be –3. Show how he could verify if his answer is correct.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOME 17. Illustrate the solution process for a two-step, single variable, first-degree equation, using concrete materials or diagrams. [CN, PS, V] (8–4)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

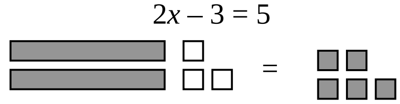
Currently Authorized Resources

- *Interactions 8*, pp. 293–297
- *Interactions 9*, pp. 232–237
- *Mathpower 8*, pp. 188–189
- *Mathpower 9*, pp. 88–91
- *Minds on Math 8*, pp. 371–376
- *Minds on Math 9*, pp. 133–138
- *TLE 8*, Linear Equations (2 Step Solution), Student Refresher pp. 46–47, Teacher's Manual pp. 104–107

Previously Authorized Resources

- *Journeys in Math 8*, p. 358
- *Journeys in Math 9*, p. 166

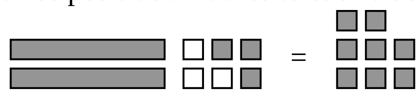
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Do not spend much time on this. It is very important for students to make the connection between what they do with the tiles and the symbolic algebra.	Using algebra tiles to demonstrate the solution of linear equations provides a visual aspect to an otherwise abstract activity. The analogy of keeping the teeter-totter balanced—doing the same thing to both sides of the equation—is very useful in promoting mathematically correct understanding of the equation-solving process. As students solve an equation, it is important that they know what their goal is: to isolate the variable. Keeping this in mind will help them know what needs to be eliminated. Students also need to understand the zero property. The following steps could be used to solve two-step linear equations. 1. Arrange the appropriate tiles to represent the equation. $2x – 3 = 5$ 

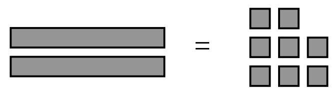
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

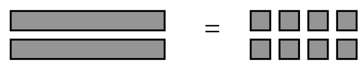
2. Eliminate the three negative unit tiles from the left side by adding three positive unit tiles to each side.



3. Remove the zero pairs from the equation.



4. Arrange the unit tiles into two equal rows—because there are two *x* tiles on the left.



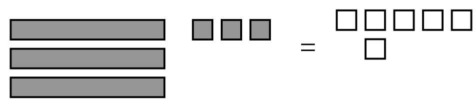
5. Remove one entire row from the equation. What remains is the answer ($x = 4$).

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

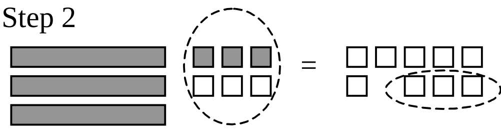
Paper and Pencil

1. Write the equivalent algebra equation for each diagram below. Describe each step in the solution of the equation.

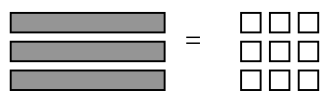
   Step 1

   Step 2

   Step 3

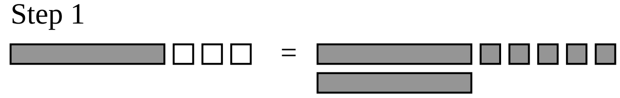
   Step 4

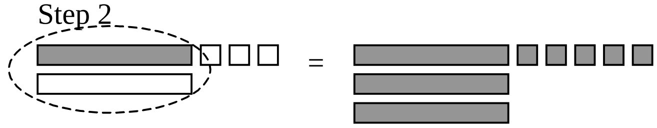
2. Use algebra tiles to solve the equations.
   a. $3x - 5 = 10$
   b. $2x - 1 = 3x + 5$
   c. $6 = 2x - 3 + x$
   d. $5x - x + 1 = 3x - 2$
   e. $-2x + 5 = -x - 3$

3. Rob used tiles to solve the equation $x - 3 = 2x + 5$. Do an error analysis on the steps he used to solve for x.

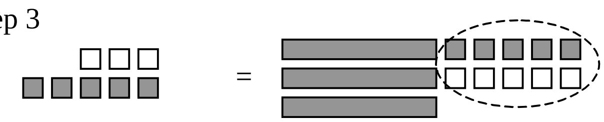
   Step 1

   Step 2

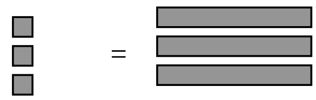
   Step 3

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

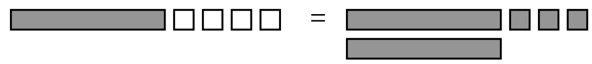
Teaching Notes

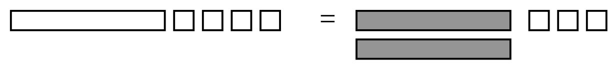
Step 4

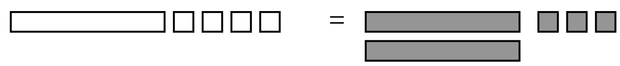


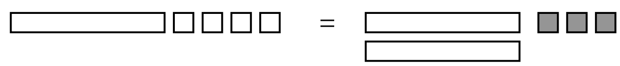
Step 5

4. Which diagram satisfies the equation $-x - 4 = 2x + 3$?

a. 

b. 

c. 

d. 

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOME

19. Solve and verify first-degree single-variable equations of the form:
    - $ax = b + cx$
    - $a(x + b) = c$
    - $ax + b = cx + d$
    - $a(bx + c) = d(ex + f)$
    - $\frac{a}{x} = b$

    where *a, b, c, d, e* and *f* are rational numbers (with a focus on integers), and use equations of this type to model and solve problem situations. [C, PS, V] (9–5)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10,* pp. 140–143, 158–161
- *Interactions 9*, pp. 232–241, 248, 250
- *Mathpower 8*, pp. 188–196
- *Mathpower 9*, pp. 88–108
- *Minds on Math 8*, p. 383
- *Minds on Math 9*, pp. 133–147, 150–159
- *TLE 9* Algebra Tiles Explorer
- *TLE 9*, Linear Equations 1–3, Student Refresher pp. 34–39, Teacher's Manual pp. 80–91

Previously Authorized Resources
- *Journeys in Math 9*, pp. 168–177, 186–189
- *Mathematics 9*, pp. 101–114

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes All substitutions should be made into parentheses in order to evaluate correctly. It may be useful to explain this with the following example: $(–2)^2 = (–2) \times (–2)$ $= 4$ $–(2)^2 = –(2 \times 2)$ $= –4$ Avoid using $–2^2$ as students often misinterpret this.	1. $ax = b + cx$ $3x = 14 - 4x$ $3x + 4x = 14 - 4x + 4x$ $7x = 14$ $\frac{7x}{7} = \frac{14}{7}$ $x = 2$ Use inverse operations to collect like terms onto one side of the equation.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Verify by substitution: $3x = 14 - 4x$ $3(2) = 14 - 4(2)$ $6 = 14 - 8$ $6 = 6$ $\therefore x = 2$ is a solution 2. $a(x+b) = c$ — Use the distributive property to simplify the left-hand side. $5(x+2) = 12$ $5x + 10 = 12$ $5x + 10 - 10 = 12 - 10$ $5x = 2$ $\frac{5x}{5} = \frac{2}{5}$ $x = \frac{2}{5}$ Verify by substitution: $5(x+2) = 12$ $5\left(\left(\frac{2}{5}\right) + 2\right) = 12$ $5\left(\frac{12}{5}\right) = 12$ $12 = 12$ $\therefore x = \frac{2}{5}$ is a solution 3. $ax + b = cx + d$ $x + 2 = 3x + 6$ $x - x + 2 = 3x - x + 6$ — Use inverse operations to collect like terms. $2 = 2x + 6$ $2 - 6 = 2x + 6 - 6$ $-4 = 2x$ $\frac{-4}{2} = \frac{2x}{2}$ $-2 = x$ Verify by substitution: $x + 2 = 3x + 6$ $(-2) + 2 = 3(-2) + 6$ $0 = -6 + 6$ $0 = 0$ $\therefore x = -2$ is a solution

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	4. $a(bx + c) = d(ex + f)$

a. $3(-2x + 4) = -4(x - 6)$

$-6x + 12 = -4x + 24$

$-6x + 6x + 12 = -4x + 6x + 24$

$12 = 2x + 24$

$12 - 24 = 2x + 24 - 24$

$-12 = 2x$

$$\frac{-12}{2} = \frac{2x}{2}$$

$-6 = x$

Simplify first, by using the distributive property.

Use inverse operations to collect like terms.

Verify by substitution:

$3(-2x + 4) = -4(x - 6)$

$3[-2(-6) + 4] = -4[(-6) - 6]$

$3(12 + 4) = -4(-12)$

$3(16) = 48$

$48 = 48$

$\therefore x = -6$ is a solution

b. $$\frac{3x-6}{8} = \frac{x+2}{4}$$

$4(3x - 6) = 8(x + 2)$

$12x - 24 = 8x + 16$

$12x - 8x - 24 = 8x - 8x + 16$

$4x - 24 = 16$

$4x - 24 + 24 = 16 + 24$

$4x = 40$

$$\frac{4x}{4} = \frac{40}{4}$$

$x = 10$

Find the cross products.

c. $$\frac{1}{3}(x+5) = \frac{2}{5}(x-1)$$

$$15\left(\frac{1}{3}\right)(x+5) = 15\left(\frac{2}{5}\right)(x-1)$$

$5(x + 5) = 3(2)(x - 1)$

$5x + 25 = 3(2)(x - 1)$

$5x + 25 = 6x - 6$

$5x - 5x + 25 = 6x - 5x - 6$

$25 = x - 6$

$25 + 6 = x - 6 + 6$

$31 = x$

$x = 31$

Multiply by the lowest common denominator.

Use the distributive property.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

In solving problems, students are asked to translate word problems into concrete models and then to symbolic equations. When students establish equations, they must:

- define the variables
- realize that word order is not necessarily the order found in the equation; e.g., the phrase "3 is subtracted from a number" implies $x - 3$
- know direct and indirect words for operations; e.g., add, sum, total.

Translating Word Problems into Symbolic Equations, and Solving the Equations

Provide students with problems such as the one below. As a class, discuss the various ways in which solutions to problems could be set up. For example, the variable could be any vehicle in question 1. Have each student create a similar problem, exchange his or her problem with a partner, solve, and then verify each other's solution.

1. There are 12 vehicles in a parking lot. There is 1 more van than trucks. There are 5 more cars than trucks. How many of each vehicle are in the parking lot?
   One Solution:

Vans	Trucks	Cars
n	$n - 1$	$(n - 1) + 5$

$$n + n - 1 + (n - 1) + 5 = 12$$
$$3n + 3 = 12$$
$$-3 = -3$$
$$3n = 9$$
$$\frac{3n}{3} = \frac{9}{3}$$
$$n = 3$$

So 3 vans, 2 trucks and 7 cars make up the 12 vehicles in the parking lot. Check: 3 + 2 + 7 = 12.

Note: Discuss other solutions where n = number of trucks and n = number of cars.

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	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Solve and verify the following equations:
   a. $5x = 12 + 2x$
   b. $7 - 2x - 3x - 1 = 21$
   c. $2(x - 3) = x + 17$
   d. $\frac{4}{x} = -2$
   e. $0.3(x + 0.2) = 2(0.1x + 0.7)$
   f. $\frac{4m}{6} - \frac{7}{2} = \frac{5m}{3}$

2. The formula $G = 2.1n + 3.7$ can be used to find how long a traffic light stays green, where G is the green time in seconds and n is the number of vehicles that proceed per light cycle.
   a. How many vehicles proceed if green time is 40 seconds?
   b. If 50 cars can proceed, how long is the green light?

3. Reed has 21 nickels and dimes totalling $1.35. How many dimes does he have?

4. The sum of three consecutive even numbers is 96. Find the numbers.