STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using perimeter, area, surface area and volume.

SPECIFIC OUTCOME

1. Estimate, measure and calculate the surface area and volume of any right prism, cylinder, cone or pyramid. [E, PS, T, R] (6–4, 7–2, 8–4)

Note: Specific outcome 1 is also addressed with specific outcome 10.

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 47–48, 76, 87, 106, 406–415
- *Interactions 8*, pp. 265–277
- *Interactions 9*, pp. 85, 88, 94, 157–158, 216–217, 286–287, 195–202
- *Mathpower 8*, pp. 248–257, 264–266
- *Mathpower 9*, pp. 266–278
- *Minds on Math 8*, pp. 420–449
- *Minds on Math 9*, pp. 444–469
- *TLE 9*, Volume and Surface Area, Student Refresher pp. 68–71, Teacher's Manual pp. 148–155

Previously Authorized Resources

- *Journeys in Math 8*, pp. 108–122
- *Journeys in Math 9*, pp. 208–215
- *Mathematics 9*, pp. 466–484

TECHNOLOGY CONNECTIONS

- Spreadsheet programs

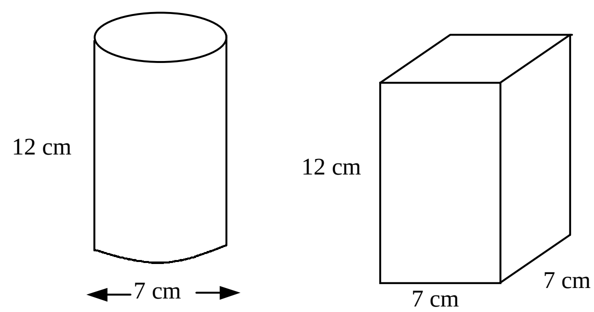
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	In order for students to have a good understanding of surface area and volume, many will need to work with actual cylinders, cones, prisms and pyramids. Measuring these objects gives meaning to the formulas that they will use to calculate surface area and volume. Also, students must be able to identify the base(s) on these objects, since many of the formulas have base as one of the components. Following are some formulas that may be used to calculate surface area and volume. The calculation of surface area often requires a combination of two or more formulas. When using formulas, proper substitution techniques should be followed. Answers should include correct units of measurement.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	$A = lw$ (area of rectangle) $A = bh$ (area of parallelogram) $A = \frac{1}{2}bh$ (area of triangle) $A = \frac{h(a+b)}{2}$ (area of trapezoid) $A = \pi r^2$ (area of circle) $V = Bh$ (volume of prism or cylinder) Note: B = area of base $V = \frac{1}{3}Bh$ (volume of pyramid or cone) $A = 2\pi r^2 + 2\pi rh$ (surface area of cylinder) $A = \pi r^2 + \pi rs$ (surface area of cone) $c^2 = a^2 + b^2$ (Pythagorean theorem) Students may use 3.14 for π, or the π button on the calculator. The π button is the preferred method. **Sample Problem** Calculate the exposed surface area and the volume of a cone-shaped pile of sand whose radius is 14 m and whose vertical height is 10.5 m. **Solution** First calculate the slant height. $c^2 = a^2 + b^2$ $c^2 = 14^2 + 10.5^2$ $c^2 = 306.25$ $c = 17.5$ Then calculate the surface area. (Exclude the base.) $A = \pi rs$ $A = (3.14)(14)(17.5)$ $A = 769.3$ The exposed surface area is 769 m^2 or 770 m^2 if the π button is used. Then calculate the volume. $V = \frac{1}{3}Bh$ $V = \frac{1}{3}(\pi r^2)h$ $V = \frac{1}{3}(3.14 \times 14^2)(10.5)$ $V = 2154.04$ The volume is 2154 m^3 or 2155 m^3 if the π button is used.

Strand: Shape and Space (Measurement)
Specific Outcome: 1. Estimate, measure and calculate the surface area and volume of any right prism, cylinder, cone or pyramid. [E, PS, T, R] (6–4, 7–2, 8–4)

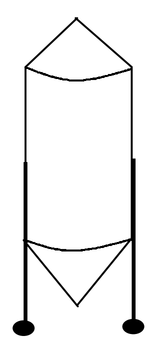
	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

1. A cardboard box is made so that a can fits snugly inside it. Both have the same height and width as shown in the diagram.

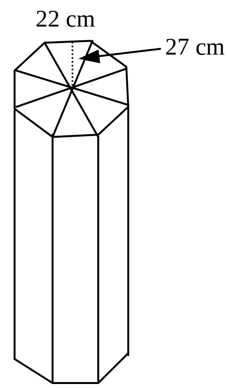
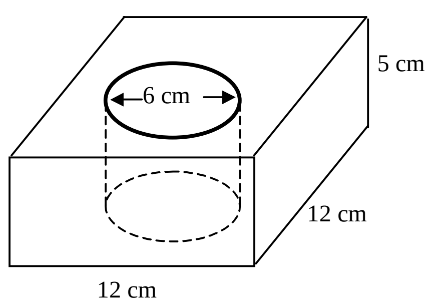


   a. Calculate the surface area of the can using the formula $A = 2\pi r^2 + 2\pi rh$.
   b. Using your answer from part a, estimate the surface area of the box.
   c. Calculate the surface area of the box, by adding the areas of each of its faces.
   d. Calculate the volume of the can, using the formula $V = \pi r^2 h$.
   e. Using your answer from part d, estimate the volume of the box.
   f. Calculate the volume of the box, using the formula $V = lwh$.

2. Many steel grain bins are made from a large cylinder, with both bases removed, and two cones, also with the bases removed. One cone is fastened at the bottom of the cylinder, and the other is fastened at the top as shown. The two cones are identical. The diameter of the cylinder is 3.2 m, and its height is 4.0 m. The cones each have a vertical height of 1.2 m and a slant height of 2.0 m.



   a. Calculate the total surface area of the grain bin.
   b. Calculate the total volume of the grain bin.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	3. An ancient stone column has the shape of an octagonal prism. It was originally used to support a roof, but the roof has fallen, and the pillar now stands alone. The top of the pillar is a regular octagon, made up of eight identical isosceles triangles, each with a base of 22 cm and a height of 27 cm. The total height of the pillar is 6.8 m.  a. Calculate the total exposed surface area of the pillar. b. Calculate the volume of the pillar. Another pillar from the same building is round instead of octagonal. It has the same height, and its radius is 27 cm. It is also standing upright, with nothing covering the top. c. Using your answer from part a, estimate the exposed surface area of the round pillar. d. Calculate the exposed surface area of the round pillar. e. Using your answer from part b, estimate the volume of the round pillar. f. Calculate the volume of the round pillar. 4. A solid block of wood measuring 12 cm by 12 cm by 5 cm has a 6 cm diameter hole drilled through it as shown. The entire block is to be painted, including inside the hole.  a. Determine the surface area to be painted. b. Determine the volume of wood in the finished product.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

It is assumed that the teacher and students are familiar with the operation of spreadsheets.

Computer Activity

1. A 355 mL aluminum pop can has a volume of 355 cm^3. Many different combinations of radius and height will result in this volume. Create a spreadsheet that will help determine which height and radius will require the least amount of aluminum (smallest surface area). Use the following column headings. Put formulas in columns C and D to calculate height and surface area from the given volume and radius.

	A	B	C	D
1	Volume	Radius	Height	Surface Area
2	355	2.0		
3	355	2.1		
4	355	2.2		
•	•	•		
•	•	•		
•	•	•		
25	355	4.3		

   a. Which height and radius combination results in the smallest surface area?
   b. How close is this to the actual size used by soft drink companies?
   c. Suggest some reasons for any differences between what the spreadsheet gives as the best combination and the measurements of an actual pop can.
   d. Modify the spreadsheet to determine the dimensions of a square-based, two-litre milk carton that will use the least amount of cardboard in its construction.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using perimeter, area, surface area and volume.

SPECIFIC OUTCOME 2. Demonstrate concretely, pictorially and symbolically that many rectangles are possible for a given perimeter or a given area. [CN, R] (6–7)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 9*, pp. 203–205, 207
- *Mathpower 8*, pp. 213, 221
- *Minds on Math 8*, pp. 305–313
- *Minds on Math 9*, pp. 450–452
- *TLE 9*, Area and Perimeter, Student Refresher pp. 72–73, Teacher's Manual pp. 156–159

Previously Authorized Resources

- *Journeys in Math 8*, p. 93

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

This outcome should not require much time. Depending on your students, you may be able to just refresh students' memories about these relationships.

1. Use a table/chart to show as many rectangles as possible with a perimeter of 30 cm and sides of whole numbers.

Diagram	*w* (cm)	*l* (cm)	$P = 2l + 2w$ (cm)
14 cm, 1 cm	1	14	30
13 cm, 2 cm	2	13	30
12 cm, 3 cm	3	12	30
11 cm, 4 cm	4	11	30
10 cm, 5 cm	5	10	30
9 cm, 6 cm	6	9	30
8 cm, 7 cm	7	8	30

a. Could there be other measurements?
b. Could we continue on with widths greater than lengths? Would it make a difference?

2. Use a table/chart to show that many rectangles can have an area of 400 cm^2.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Given that the perimeter of a rectangle is 30 cm, students could determine all possible whole number, or mixed number, dimensions of this rectangle and calculate the area for each set of dimensions. How does area vary when the perimeter is fixed? Describe the shape that gives the maximum area and the shape that gives the smallest area. 4. What happens to the area of a regular polygon as the number of sides increases? Given a perimeter of 36 cm, what is the area when the figure has 4 sides? 6 sides? 8 sides? 5. Determine the effect on the area of a polygon when its sides are doubled, tripled, quadrupled.… Can you make a rule for this? 6. Determine the effect on the area of a circle if its perimeter is doubled or halved. Can you make a rule for this?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal/Interview** 1. Can many rectangles with different lengths and widths have the same perimeter and area? 2. Can the area of a circle or square vary if the circumference or perimeter of each is fixed? Explain. 3. Is there a relationship between areas of squares and circles that have the same perimeter? Elaborate. **Paper and Pencil** 1. Find as many whole number lengths and widths for the following rectangles: a. perimeter of 70 cm b. area of 500 cm^2 c. perimeter of 195 cm d. area of 144 m^2 2. Jordan ties his dog to the basketball pole in his yard while he is at school. The rope he uses is 9 m long. What is the maximum yard area in which the dog can roam?

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME

Solve problems, using right angle triangles.

SPECIFIC OUTCOME

3. Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications. [PS] (8–2)

MANIPULATIVES

- Ruler
- Grid paper

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 25–29, 190, 388
- *Interactions 8*, pp. 246–249
- *Interactions 9*, pp. 196–200
- *Mathpower 8*, pp. 204–209
- *Mathpower 9*, p. 227
- *Minds on Math 8*, pp. 478–489
- *Minds on Math 9*, pp. 230, 310–311
- *TLE 8*, The Pythagorean Relationship, Student Refresher pp. 54–55, Teacher's Manual pp. 120–123
- *TLE 8*, Problem Solving: Using the Pythagorean Relationship, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources

- *Journeys in Math 8*, pp. 359–361
- *Journeys in Math 9*, pp. 111–114
- *Math Matters: Book 2*, pp. 54–59
- *Mathematics 9*, pp. 338–343, 468, 471, 473

TECHNOLOGY CONNECTIONS

- Scientific calculator

Teaching Notes

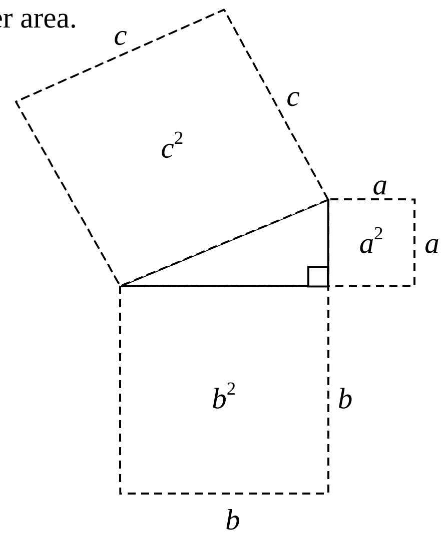
Students should also be encouraged to use rounding that is applicable to the question. For example, an answer of 152.425 km as a distance between two towns would more likely be expressed as 152 km.

Rounding by using significant digits may cause confusion, and although measurement is an important component of Applied Mathematics 10, the emphasis in this outcome should be on the use of Pythagorean theorem. Directions to students could include "round answer to the nearest tenth" or "express answer as a whole number."

Both imperial and SI units are used in Applied Mathematics 10. Examples should include both measurement systems.

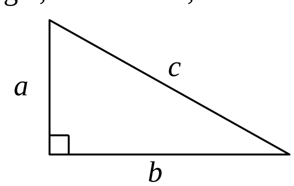
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

1. Develop the Pythagorean relationship using grid paper.
   a. Draw a right triangle.
   b. Draw a square on each side of the triangle as shown.
   c. Compare/relate areas—the two smaller areas combined give the larger area.



   Cut the squares out. Either align the larger square—on the hypotenuse—with the grid paper, or cut the smallest square into parts and fit these parts and the square on side b onto the square on the hypotenuse.

2. Pythagorean Theorem
   For any right triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse.



$c = \sqrt{a^2 + b^2}$ Finding the hypotenuse when the other two sides that make the right angle are known.

$a = \sqrt{c^2 - b^2}$

$b = \sqrt{c^2 - a^2}$

} Finding one of the shorter sides when the hypotenuse and the other side are known.

Suggestion: Students should practise rearranging the formula for the Pythagorean theorem to solve for *a* or *b*, rather than the teacher presenting individual formulas.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

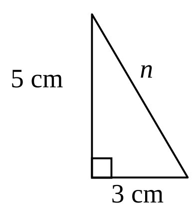
Teaching Notes

Pythagorean theorem is taught in Grade 8 and Grade 9. Less time should be spent on the actual theorem and more time on common errors, such as:

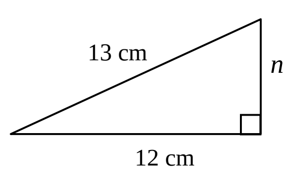
- not distinguishing the hypotenuse and always substituting the two known sides for *a* and *b* in the formula
- forgetting to find the square root—remind students that the formula solves for the square of the unknown side. Reinforcing the difference between square and square root is also important.

Students should be aware of these common triangles: 3, 4, 5; 5, 12, 13; 8, 15, 17 and any multiples of these (Pythagorean triples).

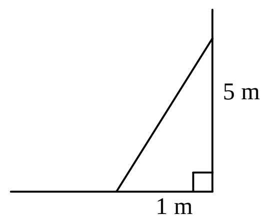
3. Examples: Find *n*. (Round to the nearest tenth.)



$$a^2 + b^2 = n^2$$
$$(3)^2 + (5)^2 = n^2$$
$$9 + 25 = n^2$$
$$34 = n^2$$
$$n = \sqrt{34}$$
$$n = 5.8 \text{ cm}$$



$$a^2 + b^2 = c^2$$
$$n^2 + (12)^2 = (13)^2$$
$$n^2 + 144 = 169$$
$$n^2 + 144 - 144 = 169 - 144$$
$$n^2 = 25$$
$$n = \sqrt{25}$$
$$n = 5 \text{ cm}$$

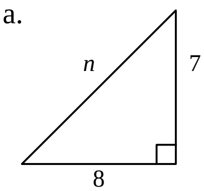
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	4. Solving Problems A ladder reaches 5 m up a wall. The bottom of the ladder is 1 m from the base of the wall. How long is the ladder? (Round to the nearest tenth.) Steps in Solution: 1. Draw diagram with information. 2. Substitute information into Pythagorean theorem. 3. Solve. 4. State answer.  $a^2 + b^2 = c^2$ $(1)^2 + (5)^2 = c^2$ $1 + 25 = c^2$ $c^2 = 26$ $c = \sqrt{26}$ $c = 5.099$ m The length of the ladder is 5.1 m, to the nearest 0.1 m.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

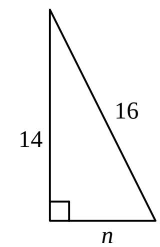
Paper and Pencil

1. Find *n*.

   a.



   b.



2. A 14 foot ladder is leaning against a tree. If the base of the ladder is three feet from the tree, how far up the tree will it reach?

3. A 40 m tower has a guy wire attached halfway up. Find the length of the guy wire if it is anchored 10 m from the base of the tower.

Journal/Interview

1. Describe the Pythagorean theorem in words.

2. Describe everyday situations that involve right angle triangles, and describe how you would find the measurements involved. Be specific; e.g., construction (mitred corner), surveying (distance that cannot be easily measured).

Portfolio

1. Collect pictures/articles from magazines and newspapers where right angle triangles appear. Measure the sides of the triangles, and show that the Pythagorean relationship is valid in every case.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME

Solve problems, using right angle triangles.

SPECIFIC OUTCOME

4. Estimate, measure and classify angles as:
   - acute
   - obtuse
   - right
   - straight
   - reflex.

   [E] (6–10)

MANIPULATIVES

- Geometry set—ruler, protractor

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 229–231
- *Mathpower 7*, pp. 264–267
- *Minds on Math 7*, pp. 368–377
- *Minds on Math 9*, pp. 183–193
- *TLE 7*, Classifying Angles, Student Refresher pp. 62–63, Teacher’s Manual pp. 136–139

Previously Authorized Resources
- *Journeys in Math 8*, pp. 236–237
- *Journeys in Math 9*, pp. 229–231
- *Math Matters: Book 2*, pp. 290–294

TECHNOLOGY CONNECTIONS

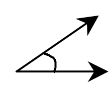
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

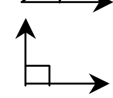
Students should have a strong intuitive understanding of the following angle measurements: 30°, 45°, 60°, 90°, 135° and 180°, and be able to draw a relatively accurate representation of each without a protractor. If this skill has been developed, then estimates of the measures of other angles will be more accurate.

1. Terminology:

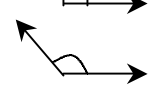
   acute angle – angle that is less than 90°; e.g., 42°

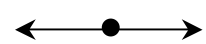
   right angle – angle that is 90°

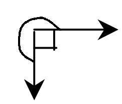
   obtuse angle – angle that is between 90° and 180°; e.g., 120°

   straight angle – angle that is 180°

   reflex angle – angle that is between 180° and 360°; e.g., 270°

2. Using a protractor and straightedge/ruler, construct a variety of angles; e.g., 47°, 116°, 90°, 285°, 180°.

3. Given a variety of angles measuring between 0° and 360°, use a protractor to find their measurement in degrees, and classify each.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Students may have difficulty finding reflex angles until they realize that the direction of rotation defines the type of angle. For example, they may perceive the corner of a door as 90°, but the angle around the corner of the door is 270°.

Interview/Journal

1. Describe an acute, right, obtuse, straight and reflex angle.

2. Look around the classroom and find an example of each angle. Which angle was easiest to find?

3. Explain how your perception of a three-dimensional object can change the classification of the angles you see. (See Teaching Notes.)

Paper and Pencil

1. Without using a protractor, draw acute, right, obtuse, straight and reflex angles; and estimate their measurement. Use a protractor to check your results.

2. Construct the following angles, using a protractor, and classify them.
   a. 174°
   b. 13°
   c. 70°
   d. 90°
   e. 240°
   f. 168°
   g. 340°

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using right angle triangles.

SPECIFIC OUTCOME 5. Explain the meaning of sine, cosine and tangent ratios in right triangles. [C, R] (9–1)

MANIPULATIVES

- Geometry set—ruler, protractor

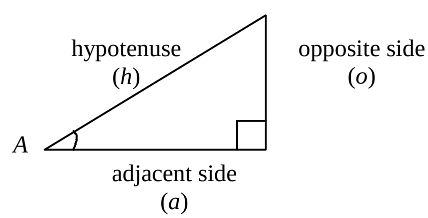
SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 487–500
- *Interactions 9*, pp. 264–265, 268–269
- *Mathpower 9*, pp. 236–244
- *Minds on Math 9*, pp. 232–239, 246–252
- *TLE 9*, Ratios in Right Triangles, Student Refresher pp. 58–59, Teacher's Manual pp. 128–131

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Trigonometric ratios are the ratios of the lengths of sides of right angle triangles. Because similar triangles have the same angles, their shape remains the same and their sides have the same ratios. The same ratios of sides, therefore, will always give the same angle. Terminology: For any given right angle triangle, the sides are labelled according to one of the given acute angles.  If the other acute angle is used, opposite and adjacent switch. sin (sine) $A = \frac{o}{h}$ cos (cosine) $A = \frac{a}{h}$ tan (tangent) $A = \frac{o}{a}$

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

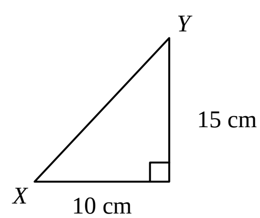
Teaching Notes

Calculators should be set in degree mode.

Graphing calculators, and many scientific calculators, use the order sin 30 to find sin 30. Other scientific calculators use 30 sin.

2. Use of Scientific Calculator with Trigonometry
   a. Given the angle → use “sin”, “cos” or “tan” key and angle measurement
      ⇒ this gives the decimal equivalent of the ratio
   b. Given the sides → put sides in appropriate ratio and divide
      ⇒ this gives the decimal equivalent of the ratio, as above
   c. Given the decimal value→ use 2^{nd} function button, appropriate trigonometry button and decimal
      ⇒ this gives the angle

3. a. Draw accurate right angle triangles with an angle of 35º and various side lengths. Make a table of these lengths, put them into the three trigonometric ratios, and compare.
   b. For a right angle triangle with short sides of 10 cm and 15 cm:
      - Find all trigonometric ratios and put them in decimal form.



$$\overline{XY} = \sqrt{15^2 + 10^2}$$
$$= 18$$

$$\sin X = \frac{15}{18} = 0.8\overline{3}$$
$$\cos X = \frac{10}{18} = 0.\overline{5}$$
$$\tan X = \frac{15}{10} = 1.5$$
$$\sin Y = \frac{10}{18} = 0.\overline{5}$$
$$\cos Y = \frac{15}{18} = 0.8\overline{3}$$
$$\tan Y = \frac{10}{15} = 0.\overline{6}$$

      - Find $\angle X$ and $\angle Y$.

$\tan X = 1.5$
$\angle X = 56.3º$

$\sin Y = 0.\overline{5}$
$\angle Y = 33.7º$
OR
$\angle Y = 90º - \angle X$
$= 90 - 56.3$
$= 33.7º$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

Journal/Interview

1. Describe the trigonometric ratios in words—what they are; i.e., ratios, and why they work as they do.

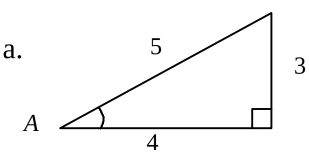
2. The unique characteristics of a right angle triangle make it useful in real-life circumstances. Support this statement.

3. Describe how you use a scientific calculator in trigonometry.

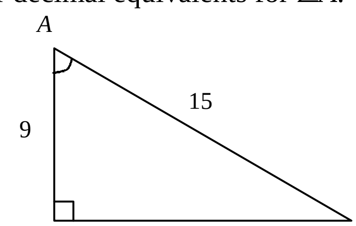
Paper and Pencil

1. Find the trigonometric ratios and their decimal equivalents for $\angle A$.

a.



b.



2. Use your calculator to determine the following:
   a. $\sin 37^\circ =$
      $\tan 85^\circ =$
      $\cos 16^\circ =$
   b. $\sin A = 0.923$ $\quad \angle A =$
      $\cos B = 0.420$ $\quad \angle B =$
      $\tan C = 2.415$ $\quad \angle C =$

3. a. Draw accurate right angle triangles with an angle of 65º and various side lengths. Make a table of these lengths and their trigonometric ratios. Compare.
   b. Draw accurate right angle triangles with sides in appropriate ratios. Make a table of trigonometric ratios, and find the angles.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using right angle triangles.

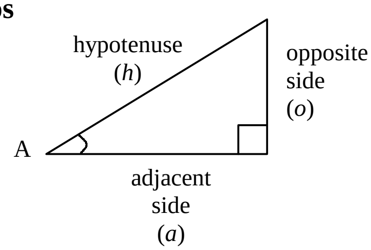
SPECIFIC OUTCOME 6. Calculate an unknown side or an unknown angle in a right triangle, using trigonometric ratios. [PS, T, V] (9–3)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

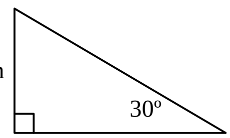
Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 487–507
- *Interactions 9*, pp. 264–277
- *Mathpower 9*, pp. 236–247, 252–255
- *Minds on Math 9*, pp. 232–263, 266–269
- *TLE 8*, The Pythagorean Relationship, Student Refresher pp. 54–55, Teacher's Manual pp. 120–123
- *TLE 8*, Problem Solving: Using the Pythagorean Relationship, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127
- *TLE 9*, Finding Unknown Sides and Angles, Student Refresher pp. 60–63, Teacher's Manual pp. 132–139

TECHNOLOGY CONNECTIONS
- Scientific calculator

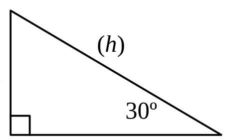
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	**Review Trigonometric Ratios** $\sin A = \frac{o}{h}$ $\cos A = \frac{a}{h}$ $\tan A = \frac{o}{a}$  Emphasize the need to name sides correctly for either acute angle. Drawing each diagram and labelling it will provide a corresponding visual. **Process and Examples** To solve right triangle problems: a. Draw a diagram and label it. b. Choose an appropriate trigonometric ratio—uses the information given. c. Substitute information into the trigonometric ratio. d. Calculate the answer.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Example 1: Find *y*.



a.



b. $\tan A = \frac{o}{a}$

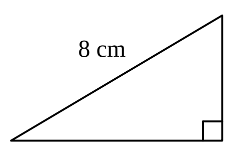
c. $\tan 30^\circ = \frac{5}{y}$

d. $y = \frac{5}{\tan 30^\circ}$

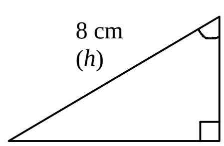
$= 8.660$

Side *y* is 8.660 cm

Example 2: Find the measure of ∠*D*.



a.



b. $\cos D = \frac{a}{h}$

c. $\cos D = \frac{6}{8}$

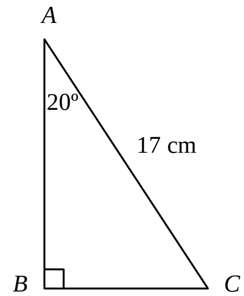
d. $\cos D = 0.75$ use 2nd function

$\angle D = 41.4^\circ$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

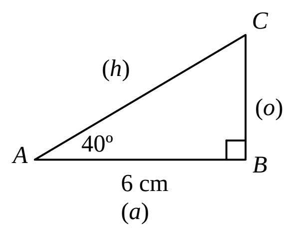
Teaching Notes

Example 3: Find the measure of $\angle C$.



Since $\angle B$ is 90º, $\angle A + \angle C = 90º$
$\therefore \angle C = 90º - 20º = 70º$

Example 4: A right angle triangle *ABC* has $\angle B = 90º$, $\angle A = 40º$ and $\overline{AB} = 6$ cm. Find the measure of $\overline{AC}$, $\overline{BC}$ and $\angle C$. Round answers to the nearest hundredth.



$\cos A = \frac{a}{h}$

$\cos 40º = \frac{6}{h}$

$h = \frac{6}{\cos 40º}$

$= 7.8324$

$\overline{AC}$ is 7.83 cm.

$\tan A = \frac{o}{a}$

$\tan 40º = \frac{o}{6}$

$o = \tan 40º \times 6$

$= 5.0346$

$\overline{BC}$ is 5.03 cm.

$\angle C = 90º - \angle A$
$\angle C = 90º - 40º$
$= 50º$
$\angle C$ is 50º.

Strand: Shape and Space (Measurement)
Specific Outcome: 6. Calculate an unknown side or an unknown angle in a right triangle, using trigonometric ratios. [PS, T, V] (9–3)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

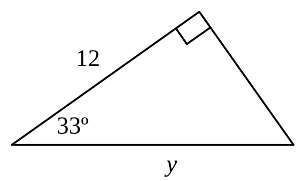
Teaching Notes

Journal/Interview

1. What is meant by opposite and adjacent sides? How are they related to the acute angles? Why is cos 60º equal to sin 30º?

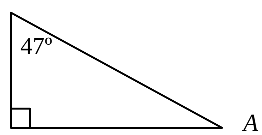
Paper and Pencil

1. Calculate *y* to three decimal places.

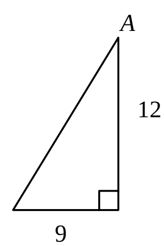


2. Find ∠*A*.

a.



b.



3. Triangle *ABC* has ∠*B* = 90º, $\overline{AB}$ = 16 cm, and $\overline{BC}$ = 11 cm. Find the measure of the missing angles and side. Round answers to the nearest hundredth of a centimetre, or to the nearest tenth of a degree.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using right angle triangles.

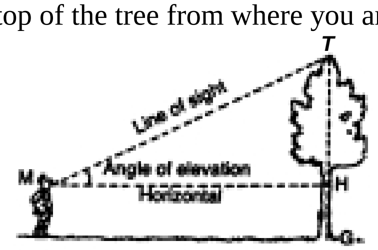
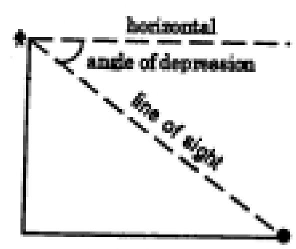
SPECIFIC OUTCOME 7. Model and then solve given problem situations involving only one right triangle. [PS, T, V] (9–4)

MANIPULATIVES
- Clinometer

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 492, 499, 500, 503–507
- *Interactions 9*, pp. 264–275
- *Mathpower 9*, pp. 238, 239, 241–247
- *Minds on Math 9*, pp. 236–239, 243–244, 249, 251, 252, 256–263
- *TLE 8*, Problem Solving: Using the Pythagorean Relationship, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127
- *TLE 9*, Solving Right Triangles, Student Refresher pp. 64–67, Teacher's Manual pp. 140–147

TECHNOLOGY CONNECTIONS

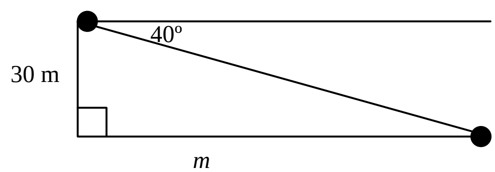
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS❶
Teaching Notes	*Angle of elevation*—If you stand and look directly at the top of a tall tree, your line of sight slopes upwards from the horizontal. The angle between your line of sight and the horizontal is called the *angle of elevation* to the top of the tree from where you are standing.  *Angle of depression*—If you stand on a cliff and look down toward an object, the angle between the horizontal and the line of sight is called the *angle of depression.* 

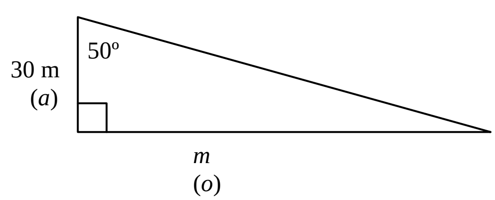
❶ The introductory Instructional Strategies/Suggestions information and Sample Questions 2 to 4 are reproduced, by permission, from Manitoba Education and Training. *Senior 1 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Surveyors measure angles of elevation with an instrument called a *theodolite*. On a more basic level, a *clinometer* can be used by students to find the height of tall structures. Clinometers can be purchased or can be constructed from a protractor, straw, string and a plumb line.

Sample Questions

1. Bill stood on a cliff overlooking a lake and saw a sailboat on the water. The cliff was 30 m high, and the angle of depression was 40º. How far out in the lake was the boat?





$$\tan A = \frac{o}{a}$$

$$\tan 50^\circ = \frac{m}{30}$$

$$\frac{\tan 50^\circ}{1} = \frac{m}{30}$$

$$m = \frac{\tan 50^\circ \times 30}{1}$$

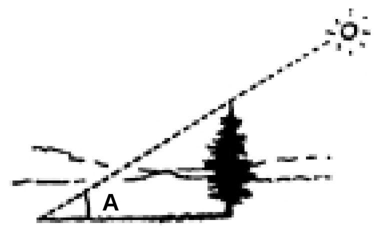
$$m = 35.7526$$

The boat was 35.75 m out in the lake.

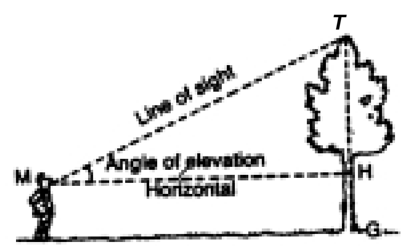
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2. A tree 2.50 m tall casts a shadow 4.36 m long. Calculate the angle of elevation of the sun to the nearest degree. (The angle is marked *A* in the diagram.)



3. Suppose that the distance *MH* in the picture is 10 m, and the angle of elevation is 25°.
   a. Calculate the distance *TH* , to the nearest 0.1 m.
   b. To find the height of the tree, you have to add the distance *HG*. This is the same as the height of the man's eyes above the ground. If his eyes are 1.6 m above the ground, how high is the tree?



   c. The same man stands 25 m away from a flagpole and finds that the angle of elevation of the top of the pole is 41°. How tall is the pole?

4. Find the height of a tall structure close to the school using a clinometer and a measuring tape.

   a. Make a scale drawing to find the height.
   b. Use trigonometric ratios to find the height.

   Compare your answers and write a detailed report on your procedures and findings.

   There is another way to complete this task. What is it? Explain.

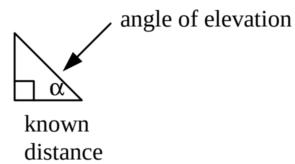
Strand: Shape and Space (Measurement)
Specific Outcome: 7. Model and then solve given problem situations involving only one right triangle. [PS, T, V] (9–4)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Projects

1. Have students measure various heights of objects without the use of a ladder; e.g., telephone poles, trees, flagpoles.



Portfolios/Journals/Interviews

1. Find examples of real-life situations that can be solved using trigonometry.

2. Find occupations that use this type of problem solving.

Paper and Pencil

1. A staircase goes to the second floor of a house that is 12 m above the ground floor. The handrail is 16 m long. What is the angle of elevation of the handrail and the horizontal distance covered?

2. A tree casts a shadow of 14.3 m. At this point the angle of elevation to the top of the tree is 27º. How tall is the tree?

STRAND: SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

GENERAL OUTCOME

Specify conditions under which triangles may be similar and use these conditions to solve problems.

SPECIFIC OUTCOME

8. Recognize when, and explain why, two triangles are similar, and use the properties of similar triangles to solve problems.
   [C, PS, R, T] (9–8)

MANIPULATIVES

- Ruler

SUGGESTED LEARNING RESOURCES

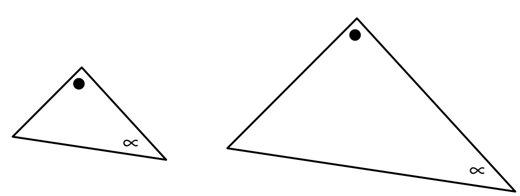
Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 480–486
- *Interactions 9*, pp. 258–262
- *Mathpower 9*, pp. 226–231
- *Mathpower 10*, pp. 226–235
- *Minds on Math 9*, pp. 208–223
- *TLE 9*, Similarity and Similar Triangles, Student Refresher pp. 74–77, Teacher's Manual pp. 160–167

Previously Authorized Resources

- *Math Matters: Book 2*, pp. 322–325
- *Mathematics 9*, pp. 390–396

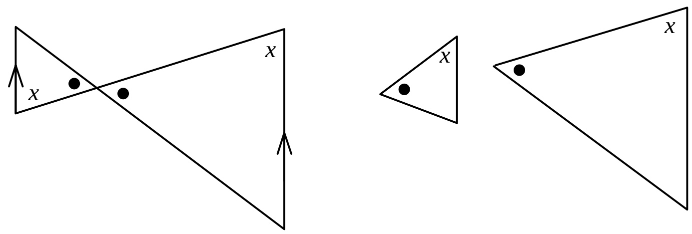
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	**Investigation** 1. Terminology: Similarity means same shape but not necessarily the same size. Similar triangles have corresponding angles that are congruent; i.e., makes the same shape, and corresponding sides that have the same ratio; i.e., proportional. If the corresponding angles of triangles are congruent, this proves similarity and makes the corresponding sides proportional. 2. Examples of similar triangles. a. Two separate triangles of the same shape. 

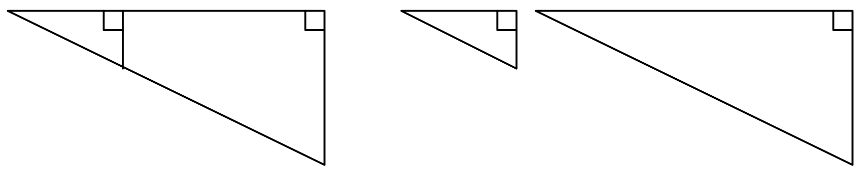
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

b. Bow tie.

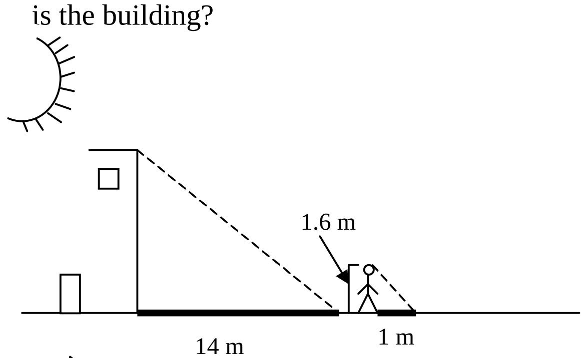


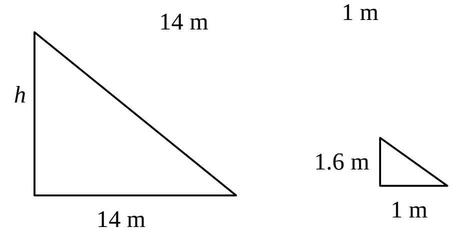
c. Triangle within a triangle.



3. Real-life problem solving where similar triangles can be used.

a. A building casts a shadow of 14 m. Tom is 1.6 m tall and casts a shadow of 1 m when standing next to the building. How tall is the building?





$$\frac{h}{14} = \frac{1.6}{1}$$

$$h = \frac{14 \times 1.6}{1}$$

$$h = 22.4$$

The building is 22.4 m tall.

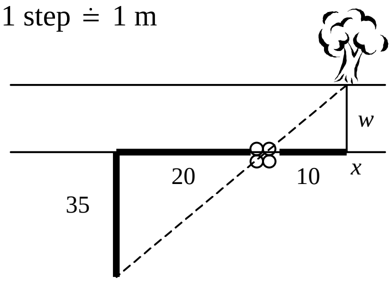
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

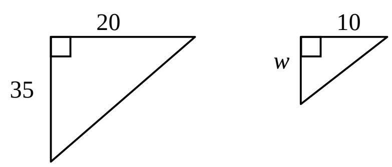
Teaching Notes

An average step is 3 feet long for a 6 foot tall person. To do parts b and c of question 3, the person should take long steps.

b. Find the width of a river.
   1. Sight a tree on the opposite bank.
   2. Turn 90º, walk along the river for 10 steps, and make a pile of stones.
   3. Continue from here for 20 steps.
   4. Make a 90º turn from the river and walk until the pile of stones and the tree line up, counting steps (say 35).
   5. You now have bow tie similarity.

   1 step $\doteq$ 1 m





$$\frac{20}{35} = \frac{10}{w}$$

$$w = \frac{35 \times 10}{20}$$

$$w = 17.5$$

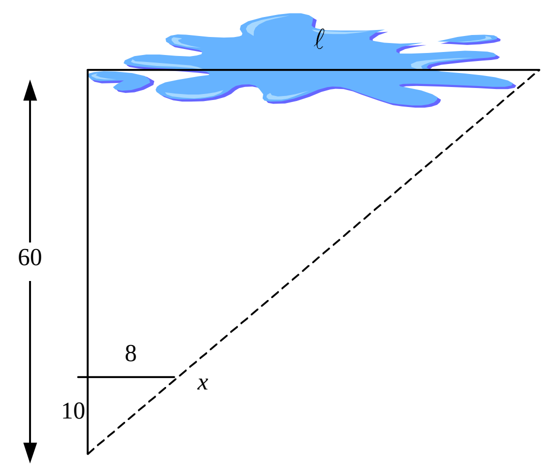
The river is 17.5 m wide.

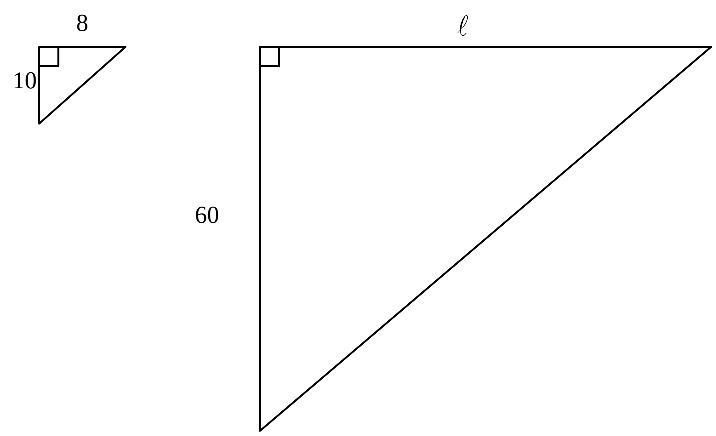
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

c. Find the length of a pond.
   1. Sight a post on the opposite side.
   2. Turn at a right angle, and walk and count very long steps until you see the post with an unobstructed view.
   3. Pound a stake into the ground at this point.
   4. Walk back along your path for 10 very long steps.
   5. Make a 90º turn right, and walk and count very long steps until the post and stake line up.
   6. You now have a triangle within a triangle.

1 step $\doteq$ 1 m





$$\frac{8}{10} = \frac{\ell}{60}$$

$$\ell = \frac{8 \times 60}{10}$$

$$\ell = 48$$

The pond is 48 m long.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Journal/Interview

1. How would you determine if two triangles are similar?

Portfolios

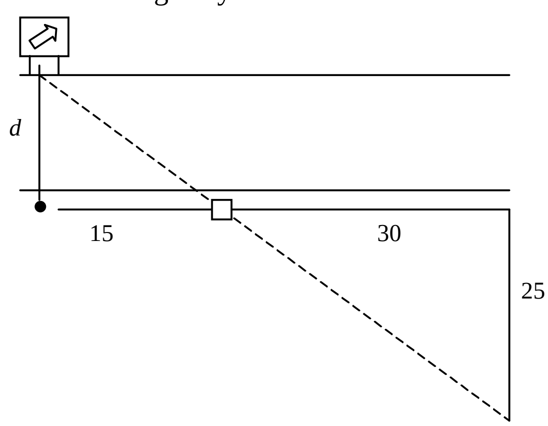
1. Go outside and measure a number of heights, using the shadow method; e.g., height of the school, flagpole, backstop, power pole.

Paper and Pencil

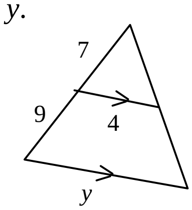
1. Answer true or false for similar triangles. Explain.
   a. They have the same shape.
   b. They have the same size.
   c. They have the same orientation.
   d. Corresponding sides are congruent.
   e. Corresponding angles are congruent.

2. A flagpole casts a shadow of 22.3 m. Bill is 1.75 m tall and casts a shadow of 3.1 m when standing next to the flagpole. How high is the flagpole?

3. Lee wants to find the distance across a highway. He sights a sign on the opposite side and then paces off distances as shown. How far is it across the highway?



4. Solve for y.



STRAND: SHAPE AND SPACE (TRANSFORMATIONS)

GENERAL OUTCOME Create and analyze patterns and design, using symmetry, translation, rotation and reflection.

SPECIFIC OUTCOME 9. Draw designs, using ordered pairs, in all four quadrants of the coordinate grid. [PS, V] (7–13)

MANIPULATIVES

- Grid paper
- Ruler

SUGGESTED LEARNING RESOURCES

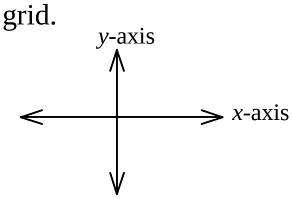
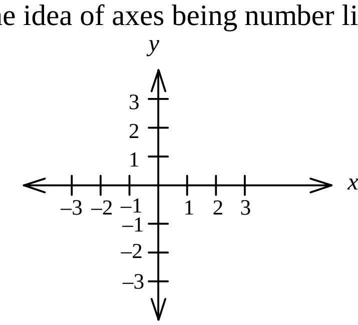
Currently Authorized Resources

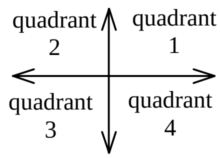
- *Interactions 7*, pp. 91–93
- *Mathpower 7*, pp. 206–211
- *Mathpower 8*, pp. 159–161
- *Minds on Math 7*, pp. 268–269
- *TLE 7*, The Coordinate Plane, Student Refresher pp. 72–73, Teacher's Manual pp. 156–159
- *TLE 9*, Transformations Explorer
- *TLE 9*, Transformations on Grids, Student Refresher pp. 86–93, Teacher's Manual pp. 184–199

Previously Authorized Resources

- *Journeys in Math 8*, pp. 278–279
- *Journeys in Math 9*, pp. 308–309
- *Math Matters: Book 2*, pp. 140–142

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Develop the terminology and vocabulary using an overhead or chalkboard grid. a.  b. Present the idea of axes being number lines with a scale of integers. 

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	c. Axes divide the grid into four parts, called quadrants.  d. Where the axes join is called the origin, and it is given the ordered pair (0, 0). e. Ordered pairs are used to locate specific points on the coordinate grid. The first number indicates a movement from the origin left (–) or right (+) and the second number indicates a movement from that point up (+) or down (–). 2. Examples Using a Grid a. Locate the following points. • *A* (3, 5) • *B* (–2, 7) • *C* (4, –1) • *D* (0, –3) • *E* $(-1, -5\frac{1}{2})$ • *F* (–7, 0) Indicate the quadrant in which they lie. b. Pick some points and have students identify their ordered pairs.

Strand: Shape and Space (Transformations)
Specific Outcome: 9. Draw designs, using ordered pairs, in all four quadrants of the co-ordinate grid. [PS, V] (7–13)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Portfolio** 1. Have students create their own picture on a grid and then develop the set of points that would produce their graphing picture. 2. Have students share their portfolio creations, and have others draw the pictures. **Journal/Interview** 1. Describe how to plot a particular ordered pair. 2. How can you always tell if an ordered pair is on: a. the *x*-axis? b. the *y*-axis? 3. Identify a way that you can determine the quadrant in which an ordered pair lies.

STRAND: SHAPE AND SPACE (TRANSFORMATIONS)

GENERAL OUTCOME

Create and analyze patterns and design, using symmetry, translation, rotation and reflection.

SPECIFIC OUTCOME

10. Draw and interpret scale diagrams:
    - enlargements
    - reductions.

    [PS, T] (8–11)

MANIPULATIVES

- Ruler
- Grid paper
- Pencil crayons/felt markers

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 321–327
- *Interactions 8*, pp. 65–72, 77
- *Interactions 9*, pp. 69–71, 73
- *Mathpower 8*, pp. 96–101
- *Mathpower 9*, pp. 309–311
- *Minds on Math 8*, pp. 134–152
- *Minds on Math 9*, pp. 406–411
- *TLE 8*, Enlargement and Reduction, Student Refresher pp. 72–73, Teacher's Manual pp. 156–159
- *TLE 9*, Dilatations and Similarity, Student Refresher pp. 90–91, Teacher's Manual pp. 192–195

Previously Authorized Resources

- *Journeys in Math 8*, pp. 182–186, 398–399
- *Journeys in Math 9*, pp. 406–411
- *Math Matters: Book 2*, pp. 205–209, 240–241

TECHNOLOGY CONNECTIONS

- *Geometer's Sketchpad*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Refer to the program of studies for an elaboration of dilatations.

Investigation Connecting Specific Outcomes 1 and 10

Problem: If the dimensions of a rectangular prism were tripled, would the volume and surface area also be tripled?

Materials:
- 3-D package; e.g., candy box, juice box
- ruler
- grid paper (3 cm × 3 cm)
- pencil crayons/felt markers

Procedure:

- Open up the package into its net.
- Measure all of its sides.
- Calculate its surface area and volume.
- Draw a 1 cm × 1 cm grid on the net.
- Enlarge this net by reproducing the net onto the 3 cm × 3 cm grid paper.
- Measure the sides of the enlarged net.
- Calculate the enlarged surface area and volume.
- Reproduce all the logos and lettering of the 1 cm × 1 cm net onto the 3 cm × 3 cm.
- Cut out the enlarged net.
- Assemble the package.

Analysis:

a. Compare your results with the class by compiling a table.

Volume	
Original Size	Enlargement

Surface Area	
Original Size	Enlargement

b. By what scale factor does volume increase, if the dimensions were tripled?

c. By what scale factor does surface area increase, if the dimensions were tripled?

Extension:

What is the relationship between the volume and surface area of an object's original and enlarged size?

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

As students are involved in the activities, observe and note if they:

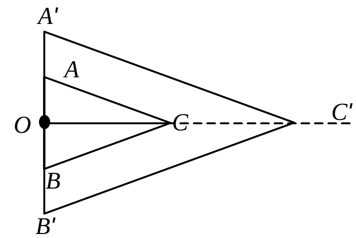
- know the geometric names of the solids, faces and geometric shapes
- measure dimensions accurately
- make reductions and enlargements accurately
- calculate surface area and volume from nets.

Sample Questions❶

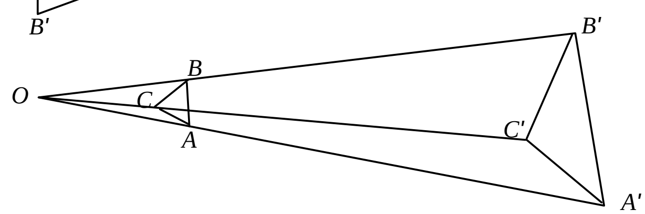
1. Give pairs of students a bucket of interlocking cubes.
   a. Show students a shape. Ask them to construct a shape which represents an enlargement by a factor of 3.
   b. Ask students to each make a shape, using exactly 6 cubes, exchange shapes with a partner, and ask their partner to create an enlargement of the shape by a factor of 2.

2. Determine the scale factor for each of the following:

   a.



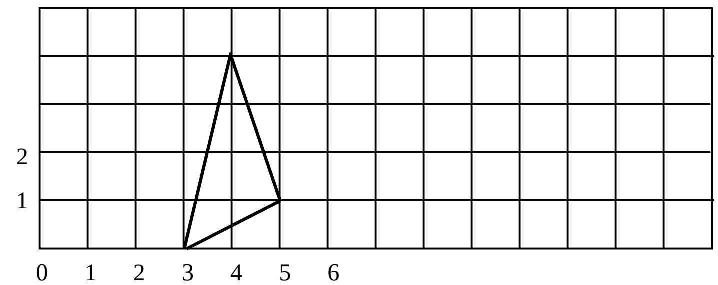
   b.



3. Reduce a figure by multiplying the *x* and *y* coordinates by the same fraction—between 0 and 1.
   a. Plot the figure and its image.
   b. Notice the differences in side lengths and areas.
   c. Notice that the shape and angles are the same.
   d. Identify the dilatation factor.

4. A photograph is 5 cm by 7 cm. It is enlarged by a scale factor of 2.5. What are the new dimensions?

❶ Sample Questions 1, 2, 4 and 5 are reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	5. In the diagram below, using the origin as the centre of dilatation, a. enlarge the given figure by a scale factor of 2 b. reduce the given figure by a scale factor of $\frac{1}{2}$ c. record the vertices for each image 

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	See below

Performance

1. Choose a cartoon to reduce in size. Draw a 3 cm × 3 cm grid onto the picture. Copy the contents of the cartoon onto 1 cm × 1 cm grid paper.

Project[1]

1. The town council of Churchill, Manitoba wants to commission a giant statue of a walking polar bear based on the sketch shown. They want a statue that is 8 m high. Your company wants to bid on making the statue. The sketch is a reduction image of a real polar bear. Measure the height of the bear in the sketch, estimate the scale factor relative to the statue, then calculate the factor. Compare with a classmate and discuss whether as a company you would use the estimated or the calculated scale factor to produce the statue. Write a report detailing to the council the size of any 3 parts of the statue, such as diameter of the eye, width of head and length of body.

Extension: The density of the statue will be the same as that of the average polar bear. Investigate the average mass of polar bears. Using this mass, calculate the cost of the statue at $55 per kilogram and add it to your report.

2. Make a 2-legged animal out of interlocking cubes. Determine its surface area and volume. What is the ratio of surface area to volume? Now double the size of the animal in all 3 dimensions. What happens to the ratio of surface area to volume?

[1] Project questions 1 and 2 are reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. a. Plot *ABCD*, where *A* = (2, 3), *B* = (–5, 3), *C* = (–5, 1) and *D* = (2, –1). Using (–2, 4) as the centre of dilatation, reduce this quadrilateral by a factor of $\frac{1}{2}$. b. Give the coordinates of the new quadrilateral *A'B'C'D'*. c. Is there a scale factor that would reduce or enlarge the quadrilateral so that it lies entirely in one quadrant? Explain your reasoning.