# *Pure Mathematics 30*

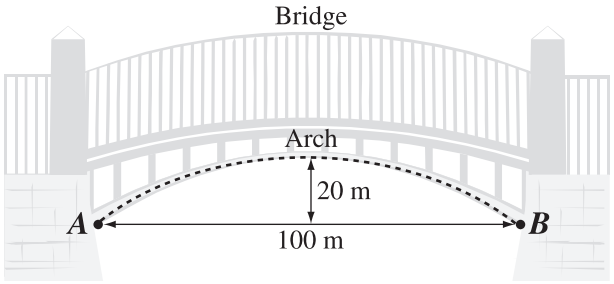
## *Project: Arch Design*

### Student Task

Engineers, architects, and designers incorporate arches in various buildings and structures for strength and/or artistic appeal. In this project, you will investigate and analyze various arch designs.

### *Part A*

Mary, a structural engineer, wants to design a curved arch support for a bridge that will have a horizontal span of 100 m and a maximum height of 20 m, similar to the diagram shown below.




**1.** As part of her design process, Mary develops some equations to model the curved arch support. Using a domain of $\{x \mid -50 \le x \le 50\}$ and a range of $\{y \mid 0 \le y \le 20\}$, determine the following types of equations that Mary could use to model the curved arch support.

- The equation of a parabola in standard form
- The equation of a semi-ellipse in general form, centre (0, 0)
- The equation of a hyperbola in the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = -1$, where $b = 10$

  **Note**: The lower arm of the hyperbola would represent the arch.

**2.** Mary decides to use either the parabolic model or the semi-elliptical model.

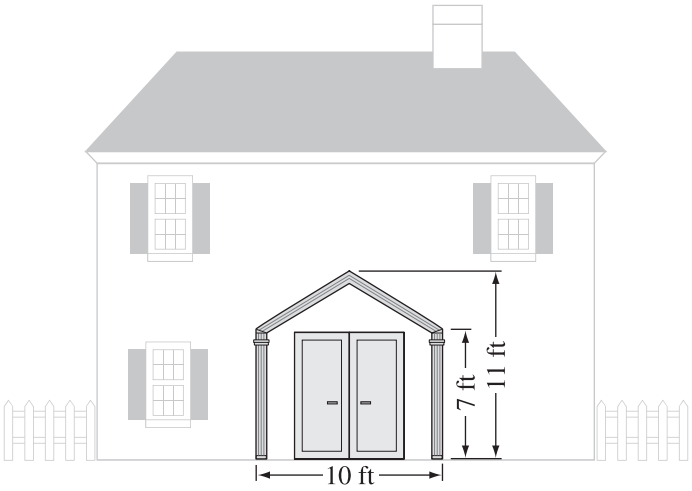
- In order to graph these models using graphing technology, you may have to rewrite their equations in the form of $y =$ _____. Write the equations you determined in question 1 in this form, and graph them on the same set of axes using the window setting $x$: $[-60, 60, 10]$, $y$: $[-5, 30, 5]$. Analyze each of these graphs and describe their similarities and differences.

**3.** The curved arch support for the bridge also needs to have a height of at least 14 m at a horizontal location of 20 m **from either end**.

- Will the parabolic model or the semi-elliptical model meet this specification? Support your answer with appropriate calculations and/or explanations.

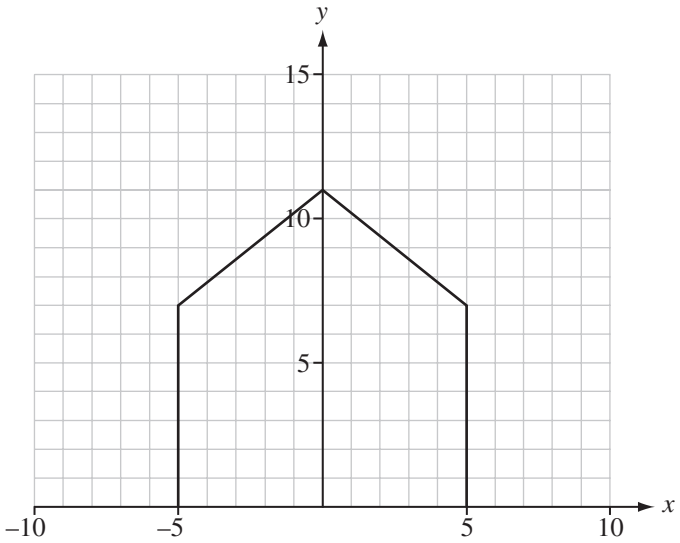
## *Part B*

John is a house designer. While he was drawing plans for a particular house, he noticed that the appearance of the house improved when a symmetrical triangular arch, supported by two pillars, was placed around the front entrance. The dimensions of the triangular arch and pillars are shown below.




An outline of the pillars and the triangular arch is shown on the grid below.

**Note:** Each square on the grid represents 1 ft × 1 ft.




The top of the triangular arch can be modelled by a single function $g(x)$ that is derived from the function $f(x) = |x|$ by a sequence of transformations, together with restrictions on the domain of $g(x)$.

- Describe in words the sequence of transformations needed to transform the graph of the function $f(x) = |x|$ into the graph of the function $g(x)$ that describes the top of the triangular arch.

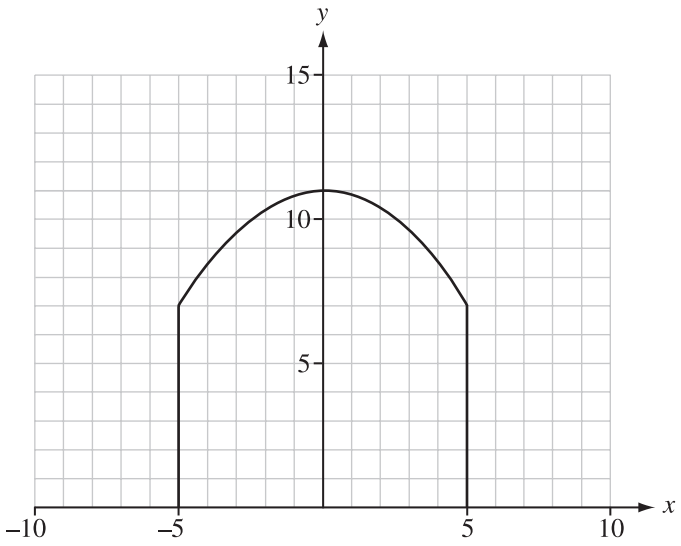
- Determine the function $g(x)$ and write it in the form $g(x) = af[b(x - h)] + k$ .

- What are the domain and range of the function $g(x)$?

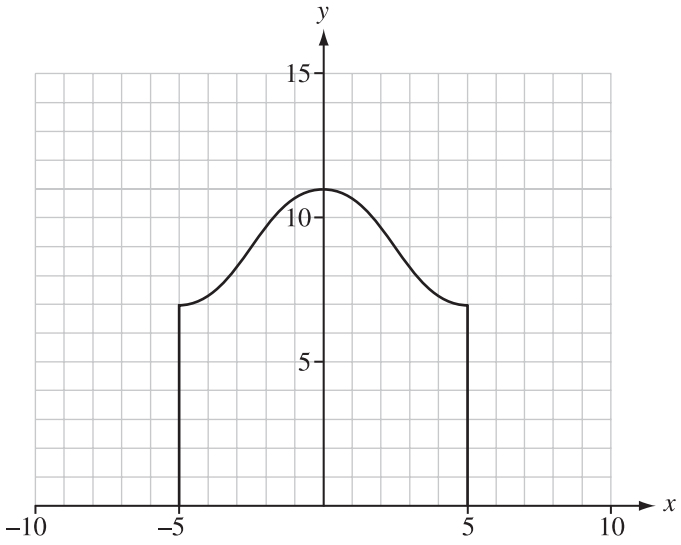
## *Part C*

As an alternative to the straight-line arches, sinusoidal wave curves could also be used. Possible designs are shown in the diagrams below.

The first diagram shows a half-wave design.




The second diagram shows a full-wave design.




**1.** For the half-wave design, determine an equation for the curve, assuming that the origin is at the point (0, 0).

**2.** For the full-wave design, determine an equation for the curve, assuming that the origin is at the point (0, 0).

*Part D*

**1.** Research catenary curves $\left(y = \frac{1}{2}\left(e^x + \frac{1}{e^x}\right) \text{ or } y = \cosh x\right)$ **or** catenary arches $\left(y = -\frac{1}{2}\left(e^x + \frac{1}{e^x}\right)\right)$ and describe the style and types of different structures that are modelled by these formulas.

**2.** On the Internet or in your community, find an example of a large bridge that contains one or more types of arch design and describe some of the specific features and characteristics of each arch (i.e., span, height, design style, weight, building process, etc.).

You may wish to use the following web sites for information:

**Arches**
www.cmhpf.org/kids/Guideboox/GothicArchitecture.html

**Bridge Arches**
http://www.ci.minneapolis.mn.us/about/stonearch.asp
http://www.calgarypubliclibrary.com/calgary/historic_tours/corner/centre.htm

**Catenary and Suspension Bridges**
http://www.humberbridge.co.uk/technical.php
http://www.icomos.org/studies/bridges.htm#13

**Elliptical Arches**
http://www.web40571.clarahost.co.uk/Strand/KewBridge.htm

**Natural Arches**
http://www.naturalarches.org/gallery.htm

Using a search engine, type any of the following terms to access a variety of sites:
"bridge arches"
"arch design"
"parabolic arches"
"elliptical arches"
"catenary curves"
"catenary arches"