January 2002

Pure Mathematics 30

Grade 12 Diploma Examination

Alberta
LEARNING

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Pure Mathematics 30

Grade 12 Diploma Examination

Description

Time: This examination was developed to be completed in 2.5 h; however, you may take an additional 0.5 h to complete the examination.

This is a **closed-book** examination consisting of

- 33 multiple-choice and 6 numerical-response questions, of equal value, worth 65% of the examination
- 3 written-response questions worth 35% of the examination

A tear-out formula sheet and two *z*-score pages are included in this booklet.

All graphs on this examination are computer-generated.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*

Instructions

- You are expected to provide a graphing calculator approved by Alberta Learning.
- You are expected to have cleared your calculator of all information that is stored in the programmable or parametric memory.
- Use only an HB pencil for the machine-scored answer sheet.
- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- Read each question carefully.
- Consider all numbers used in the questions to be **exact** numbers and not the result of a measurement.
- If you wish to change an answer, erase **all** traces of your first answer.
- Do not fold the answer sheet.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Learning.
- Now turn this page and read the detailed instructions for answering machine-scored and written-response questions.

Multiple Choice

- Decide which of the choices **best** completes the statement or answers the question.

- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

Example

This examination is for the subject of

A. biology
B. physics
C. chemistry
D. mathematics

Answer Sheet

Ⓐ Ⓑ Ⓒ ●

Numerical Response

- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.

- If an answer is a value between 0 and 1 (e.g., 0.7), then be sure to record the 0 before the decimal place.

- **Enter the first digit of your answer in the left-hand box. Any boxes on the right that are not needed are to remain blank.**

Examples

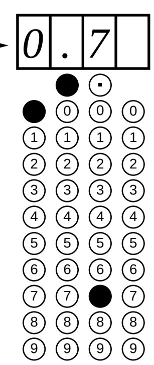
Calculation Question and Solution

Correct to the nearest tenth of a radian, 40° is equal to __________ rad.

$$40^\circ = 0.6981317008\ldots \text{ rad}$$
$$\doteq 0.7$$

(Record your answer in the numerical-response section on the answer sheet.)

Record 0.7 on the answer sheet →



Correct-Order Question and Solution

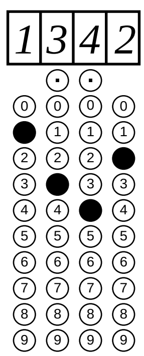
When the following subjects are arranged in alphabetical order, the order is ______ , ______ , ______ , and ______ .

1 biology
2 physics
3 chemistry
4 mathematics

(Record all **four digits** of your answer in the numerical-response section on the answer sheet.)

Answer: 1342

Record 1342 on the answer sheet →



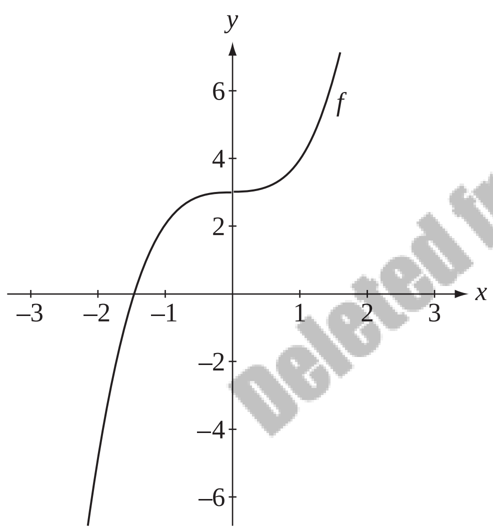
Written Response

- Write your responses in the examination booklet as neatly as possible.
- For full marks, your responses must address **all** aspects of the question.
- Descriptions and/or explanations of concepts must include pertinent ideas, diagrams, calculations, and formulas.
- Your responses must be presented in a well-organized manner using complete sentences and correct units.

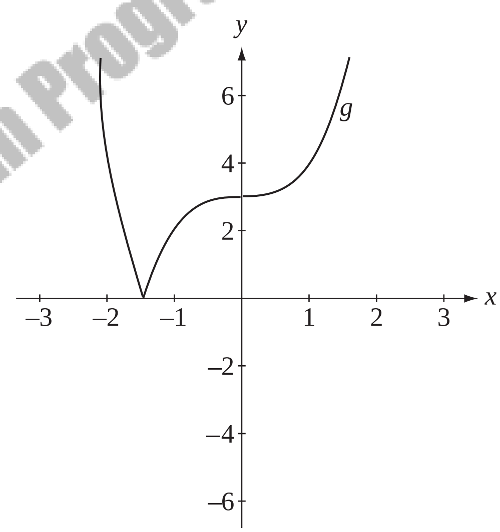
Use the following information to answer the first question.

The partial graph of a cubic function, f, is shown on the left, and the partial graph of a transformation of f is shown on the right.

Partial Graph of
$y = f(x)$



Partial Graph of $y = g(x)$,
the Transformation of $y = f(x)$



1. Which of the following transformations of f results in graph g shown on the right?

A. $g(x) = \dfrac{1}{f(x)}$

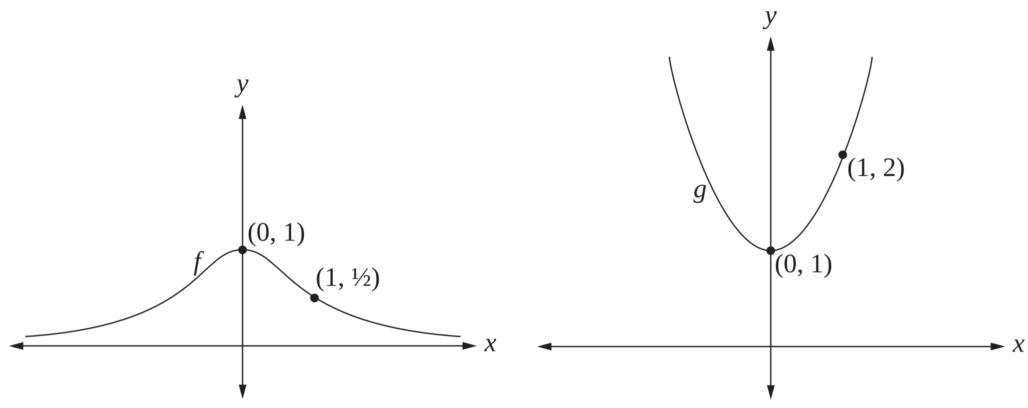
B. $g(x) = |f(x)|$

C. $g(x) = \dfrac{1}{|f(x)|}$

D. $g(x) = \left|\dfrac{1}{f(x)}\right|$

Use the following information to answer the next question.

The partial graphs of two functions, f and g, are shown below.



2. If graph g is a transformation of graph f, then the equation that would generate graph g is

A. $g(x) = f(-x)$

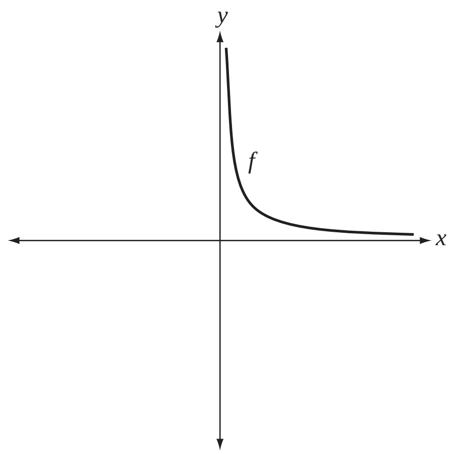
B. $g(x) = f^{-1}(x)$

C. $g(x) = -f(x)$

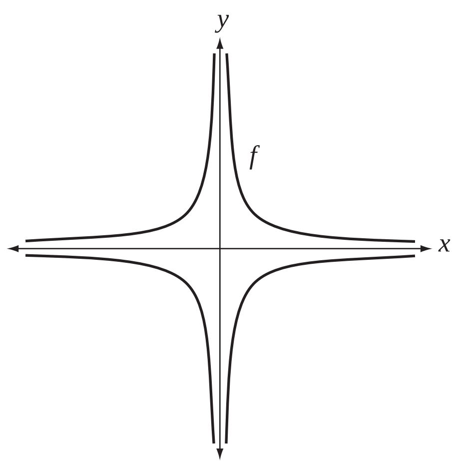
D. $g(x) = \dfrac{1}{f(x)}$

Use the following information to answer the next question.

The graph of function $y = f(x)$ is shown below.



The design below was produced by using f and three different transformations of f.



The equations related to the three transformations are

1 $y = f(-x)$
2 $y = -f(x)$
3 $y = -f(-x)$

Numerical Response

1. The equation that produced the portion of the design found in

quadrant II is equation number ____________ (Record in the first column)
quadrant III is equation number ____________ (Record in the second column)
quadrant IV is equation number ____________ (Record in the third column)

(Record all **three digits** of your answer in the numerical-response section on the answer sheet.)

3. The graph of $y = f(x) = b^x$, where $b > 1$, is translated such that the equation of the new graph is expressed as $y - 2 = f(x - 1)$. The range of the new function is

A. $y > 2$

B. $y > 3$

C. $y > -1$

D. $y > -2$

4. If y is replaced by $\frac{1}{2}y$ in the equation $y = f(x)$, then the graph of $y = f(x)$ will be stretched

A. horizontally about the y-axis by a factor of $\frac{1}{2}$

B. horizontally about the y-axis by a factor of 2

C. vertically about the x-axis by a factor of $\frac{1}{2}$

D. vertically about the x-axis by a factor of 2

5. The graph of $y = x^3$ was transformed to the graph of $y = (x - 3)^3 + 4$. Which of the following statements describes the transformation?

A. The graph of $y = x^3$ has been translated 4 units to the right and 3 units upward.

B. The graph of $y = x^3$ has been translated 3 units to the left and 4 units downward.

C. The point (x, y) on the graph $y = x^3$ has been translated to point $(x + 3, y + 4)$.

D. The point (x, y) on the graph $y = x^3$ has been translated to point $(x - 3, y - 4)$.

6. In the first stage of a letter-writing campaign, a person writes the same letter to each of 3 pen pals. In the second stage, each of these 3 pen pals copies the letter and sends it to 3 other pen pals, and so on, following this pattern. If it is assumed that no pen pal receives a letter twice, the least number of stages it would take until the minimum total sum of pen pals who received the letter is 9 800 is

A. 7

B. 8

C. 9

D. 10

7. A new coal mine produced 8 Mt (megatonnes) of coal in 1997, 7 Mt in 1998, and 6.125 Mt in 1999. If this geometric pattern were to continue indefinitely, then the total mass of coal produced by this mine would approach

A. 21.125 Mt

B. 64 Mt

C. 156.9 Mt

D. an infinite mass

Numerical Response

2. If $\log_7 m = -\frac{2}{3}$, then the value of m, correct to the nearest hundredth, is __________.

(Record your answer in the numerical-response section on the answer sheet.)

8. In 1996, a particular car was valued at $27 500 and its value decreased exponentially each year afterward. For each of the first 7 years, the value of the car decreased by 24% of the previous year's value. If t is the number of years and v is the value of the car, then the equation for the car's value when $t \leq 7$ is

A. $v = 27\ 500(1.76)^t$

B. $v = 27\ 500(1.24)^t$

C. $v = 27\ 500(0.76)^t$

D. $v = 27\ 500(0.24)^t$

Use the following information to answer the next question.

The equation that defines the decibel level for any sound is

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$ where L = loudness in decibels

I = intensity of sound being measured

I_0 = intensity of sound at the threshold of hearing

9. Given that normal conversation is 1 000 000 times as intense as I_0, then the loudness of normal conversation is

A. 5 decibels

B. 6 decibels

C. 16 decibels

D. 60 decibels

Use the following information to answer the next question.

The relationship between the length and mass of a particular species of snakes is

$$\log m = \log a + 3 \log l$$

where a is a given constant
m is the mass of the snake in grams
l is the length of the snake in metres

10. This relationship can also be written as

A. $\log\left(\dfrac{m}{al^3}\right) = 0$

B. $\log\left(\dfrac{m}{3al}\right) = 0$

C. $\log\left(\dfrac{am}{l^3}\right) = 0$

D. $\log\left(\dfrac{am}{3l}\right) = 0$

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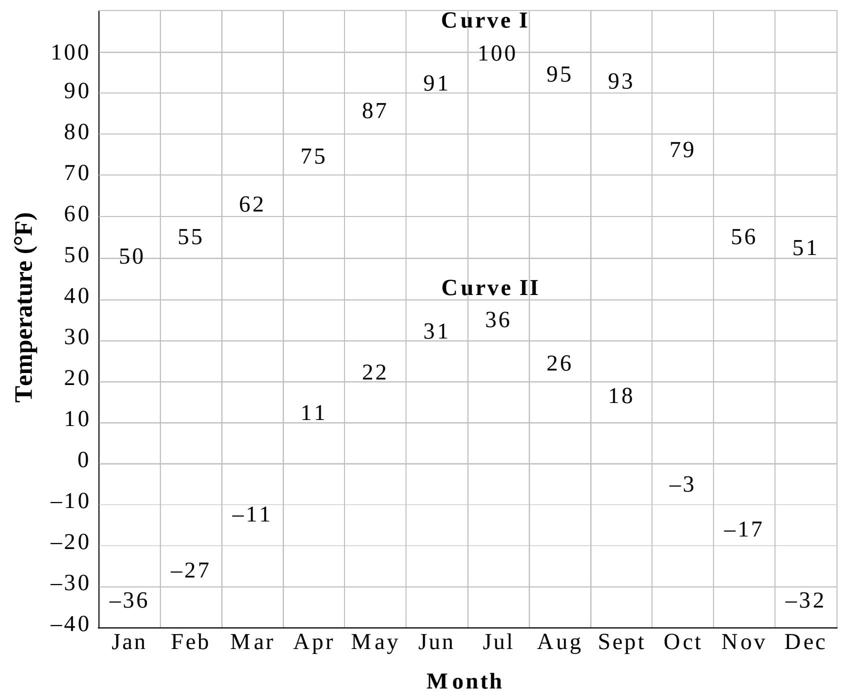
11. The x-intercept of the graph of $y = \log_b x$, where $b > 0$ and $b \neq 1$, is

A. 0
B. 1
C. undefined
D. dependent on the value of b

Use the following information to answer the next question.

The two sets of numbers shown on the graph below represent the highest and lowest temperatures over 12 months at Waterton National Park in southern Alberta.



12. Each of these sets of numbers can be modelled by a sinusoidal curve of the form $y = a \sin [b(t - c)] + d$. The two parameters in the equation representing curve I that would change the most in value in the equation representing curve II are

A. a and b

B. b and c

C. c and d

D. a and d

13. The value of $\cos^2\left(\frac{\pi}{6}\right) - \sin^2\left(\frac{\pi}{6}\right)$ is equal to the value of

A. $\cos\frac{\pi}{12}$

B. $\cos\frac{\pi}{6}$

C. $\cos\frac{\pi}{3}$

D. $\cos\frac{\pi}{2}$

14. In the equation $5\sin(2x) + 2 = \cos(x) - 1$, the number of solutions for x, where $0 \le x < 2\pi$, is

A. 2
B. 3
C. 4
D. 5

15. When the following pairs of functions are graphed, the pair that could **not** be used to solve the equation $4\sin x - 1 = 0$ is

A. $y = \sin x$ and $y = 1$

B. $y = \sin x$ and $y = \frac{1}{4}$

C. $y = 4\sin x$ and $y = 1$

D. $y = 4\sin x - 1$ and $y = 0$

16. Three consecutive terms of a geometric sequence are $\tan\theta$, $2\sin\theta$, and $4\sin\theta\cos\theta$. The common ratio of this sequence is

A. 2

B. $\cos\theta$

C. $2\cos\theta$

D. $\frac{1}{2}\sec\theta$

17. The expression $\sin\theta + (\cot\theta)(\cos\theta)$ is equivalent to

A. 1
B. $\csc\theta$
C. $\sec\theta$
D. $\cot^2\theta$

18. Which of the following statements does **not** describe the graph of $f(\theta) = -3\sin\left(\theta - \frac{\pi}{2}\right)$?

A. The amplitude is 3.

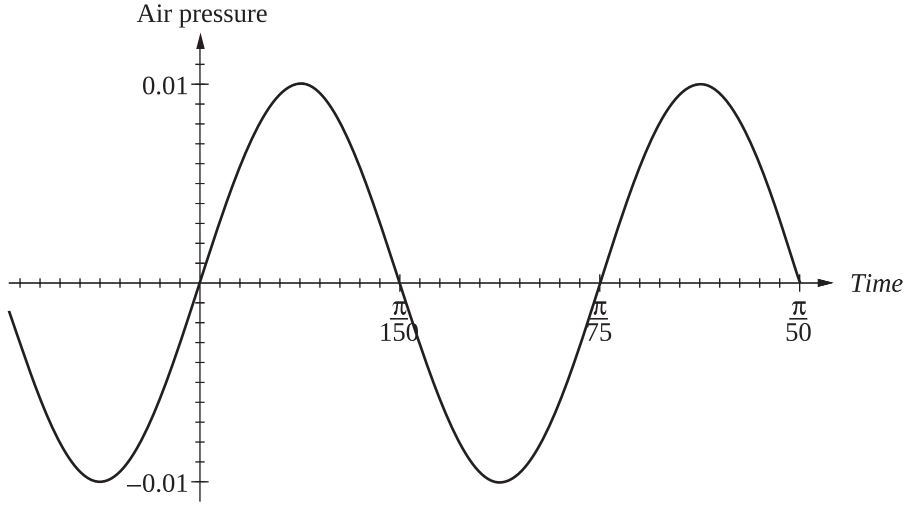
B. The period is 2π.

C. The graph of $f(\theta) = -3\sin\left(\theta - \frac{\pi}{2}\right)$ is the same as the graph of $f(\theta) = -3\sin(\theta)$ with a phase shift of $\frac{\pi}{2}$ to the right.

D. The graph of $f(\theta) = -3\sin\left(\theta - \frac{\pi}{2}\right)$ is the same as the graph of $f(\theta) = \sin\left(\theta - \frac{\pi}{2}\right)$ with a vertical translation of 3 units down.

Use the following information to answer the next question.

A computer-generated partial graph of the air pressure variation of a piano's lowest note is illustrated below. The intercepts shown are at $0, \frac{\pi}{150}, \frac{\pi}{75}$, and $\frac{\pi}{50}$.



Numerical Response

3. Given that pitch, which is measured in Hertz (Hz), is the reciprocal of the period, the pitch of this note, correct to the nearest tenth, is __________ Hz.

(Record your answer in the numerical-response section on the answer sheet.)

19. The range of the ellipse $\frac{x^2}{81} + \frac{y^2}{25} = 1$ is

A. $-9 \le y \le 9$
B. $-5 \le y \le 9$
C. $-9 \le y \le 5$
D. $-5 \le y \le 5$

20. When converted to general form, the equation $\frac{(x-3)^2}{2} - \frac{(y+2)^2}{3} = 1$ is

A. $3x^2 - 2y^2 - 18x - 8y + 13 = 0$
B. $3x^2 - 2y^2 - 18x - 8y + 19 = 0$
C. $3x^2 - 2y^2 - 18x + 8y + 29 = 0$
D. $3x^2 - 2y^2 - 18x + 8y + 35 = 0$

Numerical Response

4. The conic described by $\frac{x^2}{18} + \frac{y^2}{9} = 1$ will have a positive x-intercept at $(m\sqrt{n}, 0)$, where m and $n \in N$ and $m > 1$. The values of m and n are, respectively, __________ and __________.

(Record your answer in the numerical-response section on the answer sheet.)

21. A right circular cone is intersected by a plane that is parallel to a generator of the cone. The shape of the conic section formed in the plane is

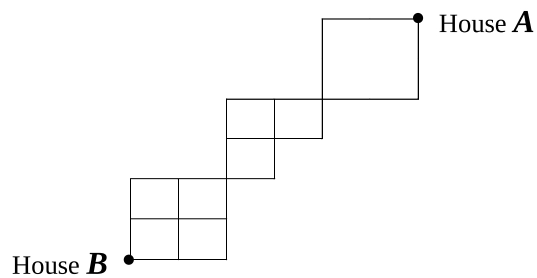
A. a hyperbola
B. a parabola
C. an ellipse
D. a circle

22. The equation of the conic $\frac{(x-3)^2}{25} - \frac{(y+2)^2}{9} = 1$ represents a hyperbola with

A. centre at (3, –2) and vertices at (3, 3) and (3, –7)
B. centre at (–3, 2) and vertices at (–3, 3) and (–3, –7)
C. centre at (–3, 2) and vertices at (8, 2) and (–2, 2)
D. centre at (3, –2) and vertices at (8, –2) and (–2, –2)

Use the following information to answer the next question.

A paperboy who delivers papers on his bike can travel only on the trails represented in the diagram below.



23. The number of different trails that the paperboy can take to get from house ***A*** to house ***B*** without backtracking is

A. 13
B. 32
C. 60
D. 72

24. The number of different arrangements of 3 boys and 4 girls in a row, if the girls **must** stand together, is represented by

A. $4! \times 4!$

B. $3! \times 4!$

C. $4! \times 4! \times 2!$

D. $3! \times 4! \times 2!$

25. The students in a music department have practised 6 contemporary and 5 traditional choruses. For their concert, they will choose a program in which they present 4 of the contemporary and 3 of the traditional choruses. How many different programs can be presented, if the order of the choruses does **not** matter?

A. 25

B. 35

C. 150

D. 330

26. All telephone numbers are preceded by a 3-digit area code. In the original Bell Telephone System of assigning area codes, the first digit could be any number from 2 to 9, the second digit was either 0 or 1, and the third digit could be any number except 0. In this system, the number of different area codes possible was

A. 126

B. 144

C. 160

D. 576

Numerical Response

5. A term of the binomial expansion $(ax + y)^8$, where $a > 0$, is $112x^2y^6$. The value of a, correct to the nearest whole number, is __________.

(Record your answer in the numerical-response section on the answer sheet.)

27. On an English test, the class mean was 40% with a standard deviation of 5%. The teacher decided to add an additional 20% to each student's test score. When the teacher recalculated the mean and standard deviation, he found that

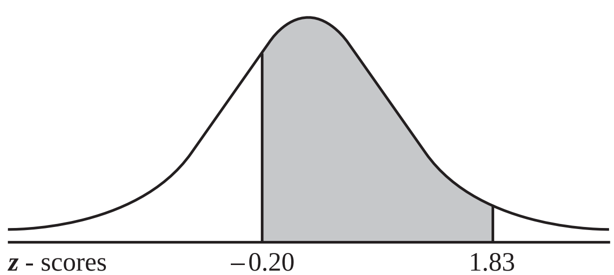
A. both the mean and standard deviation had increased
B. both the mean and standard deviation remained unchanged
C. the mean remained unchanged and the standard deviation had increased
D. the mean had increased and the standard deviation remained unchanged

28. Test scores for a particular examination are normally distributed with a mean of 67.4% and a standard deviation of 10.5%. What is the probability of a student getting less than 80% on the test?

A. 0.88
B. 0.51
C. 0.38
D. 0.12

Use the following information to answer the next question.

A standard normal distribution is shown below.



29. The area of the shaded region, correct to the nearest hundredth, is

A. 0.39
B. 0.55
C. 1.63
D. 2.03

30. The mean examination mark of a random sample of 225 students is 64.0%, with a standard deviation of 7.5%. The symmetric 95% confidence interval for individual student marks in this sample is from

A. 49.3% to 78.7%

B. 63.2% to 64.8%

C. 63.5% to 65.5%

D. 64.0% to 78.7%

Numerical Response

6. A baseball pitcher has 65% of his pitches called strikes. For the next 200 pitches he throws, the expected standard deviation of the number of those pitches that will be called strikes, correct to the nearest tenth, is __________.

(Record your answer in the numerical-response section on the answer sheet.)

31. From two vases that each contain purple and yellow tulips, a florist randomly selected two tulips. If the selection constituted an independent event, then the florist must have selected

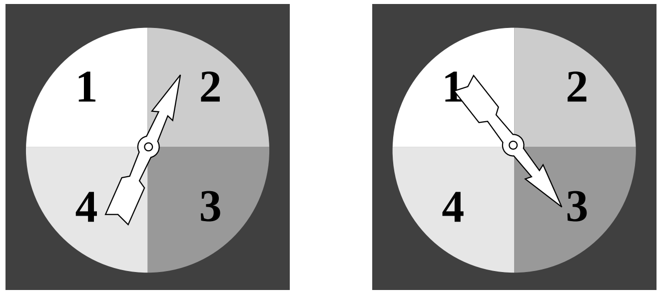
A. one tulip from the first vase and one tulip from the second vase

B. one purple tulip and one yellow tulip from the second vase

C. two tulips of the same colour from the first vase

D. both tulips from the same vase

Use the following information to answer the next question.



Two identical spinners each have four equal areas. Each arrow is spun once and stops in one of the numbered areas.

32. The probability that the sum of the two numbered areas in which the arrows stops is greater than 4 is

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{5}{8}$

D. $\frac{13}{16}$

Use the following information to answer the next question.

At a family reunion, door prizes are to be given out. At one table in the community hall, 6 children, 3 teenagers, 4 adults, and 5 seniors are seated. The 3 winning tickets are held by 3 different people at this table.

33. The probability that the 3 winning tickets are held by 3 people in the same age group, correct to the nearest thousandth, is

A. 0.037

B. 0.043

C. 0.222

D. 0.980

Written Response—10%

1. Two students, Peter and Ken, performed a simulation that involved the tossing of 3 coins. They recorded the number of heads and tails from 10 trials, as shown below.

Trial 1	HHT	Trial 6	THH
Trial 2	HHT	Trial 7	HTH
Trail 3	HTT	Trial 8	TTT
Trial 4	HHH	Trial 9	HHH
Trial 5	HTT	Trial 10	TTT

- According to the results from the simulation above, what is the experimental probability of getting 3 heads on a single trial?

Use the following information to answer the next part of the question.

Ken stated that the experimental probability of getting heads in this simulation was higher than he had expected. He defined the sample space as HHH HHT HTH HTT TTT TTH THT THH Ken reasoned that because there were 8 possible outcomes, the probability of getting 3 heads on a single trial was $\frac{1}{8}$.	Peter suggested that the sample space should be 3 heads 2 heads and 1 tail 3 tails 2 tails and 1 head Peter, reasoned that because there were only 4 possible outcomes, the probability of getting 3 heads on a single trial was $\frac{1}{4}$.

- Determine who was correct and justify your choice.

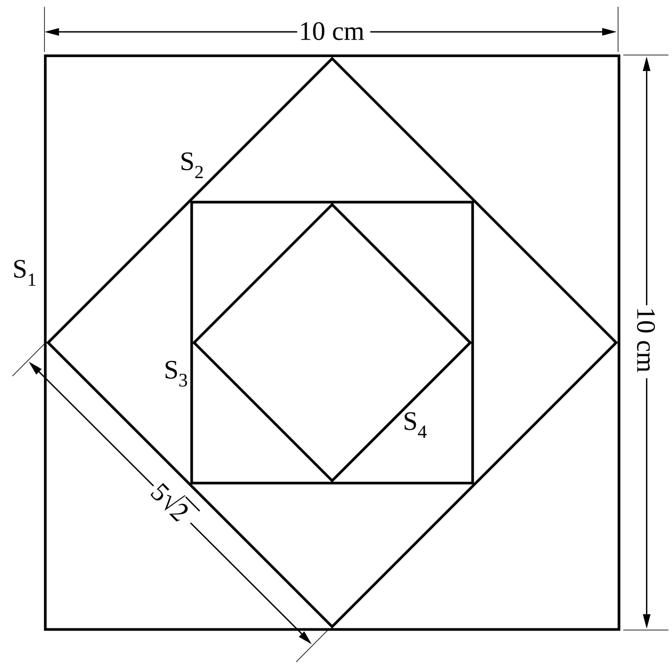
- Use the formula ${}_nC_k\, p^k(1-p)^{n-k}$ and the sample space that **you chose** in the previous bullet to determine, to the nearest thousandth, the theoretical probability of flipping exactly 2 sets of HHH in 10 trials.

- Ken suggests that the probability of flipping exactly 2 sets of HHH in 10 trials can be modelled using a normal approximation of the binomial distribution.
  Is Ken correct? Justify your answer.

Written-response question 2 begins on the next page.

Use the following information to answer the next question.

A square, S_1, with sides of 10 cm is modelled below. A second square, S_2, with sides of $5\sqrt{2}$ cm is inscribed in S_1 so that the vertices of S_2 lie at the midpoints of the sides of S_1. A third square, S_3, is inscribed in S_2 so that the vertices of S_3 lie at the midpoints of the sides of S_2. This process is continued indefinitely.



Written Response—15%

2. • The lengths of the sides of $S_1,\ S_2,\ S_3,\ \ldots$ form a geometric sequence. Determine the exact length of a side of S_3.

• The **perimeter** of each square forms a term in a geometric sequence. Determine the **exact** value of the **perimeter** of S_4.

Written-response question 2 continues on the next page.

- If the process of drawing the squares were to continue indefinitely, then what would be the sum of the infinite geometric series of the squares' **perimeter** values, correct to the nearest hundredth of a centimetre?

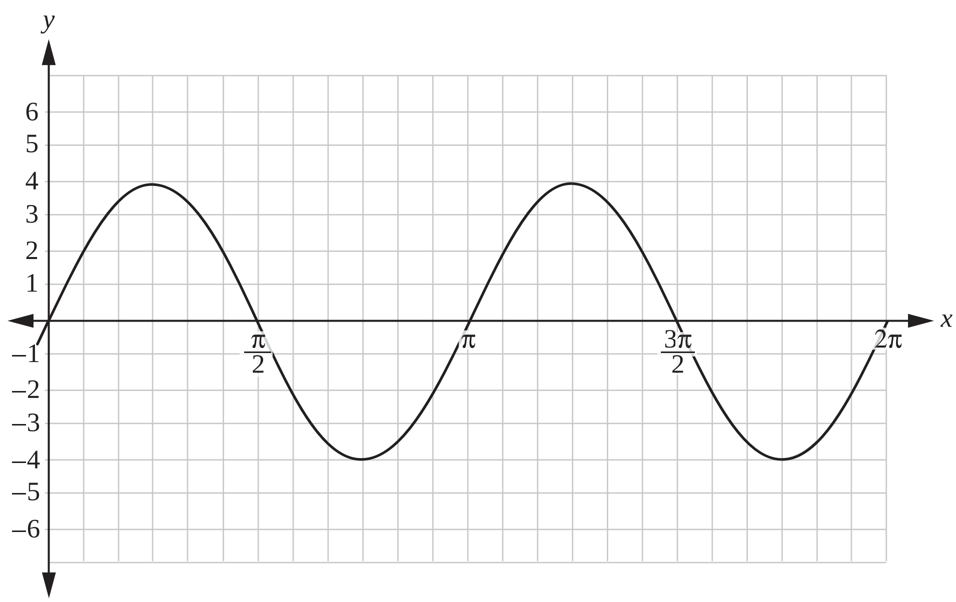
- Show how the **areas** of consecutive squares form a sequence that is geometric.

- Determine an expression that represents the sum of the first k terms of the geometric sequence calculated in the previous bullet.

Written-response question 3 begins on the next page.

Use the following information to answer the next question.

The partial graph of $f(x) = 8 \cos x \sin x$ is shown below.



Written Response—10%

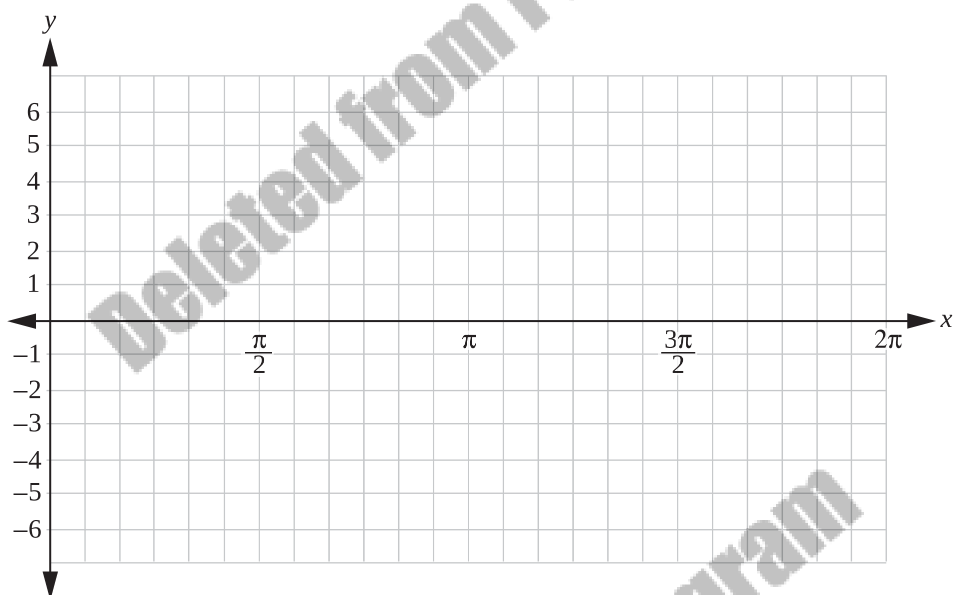
3. • Identify the amplitude and the period of f.

• Determine an equation in the form of $g(x) = a \sin[b(x - c)] + d$ that defines f as a sine function.

Written-response question 3 continues on the next page.

• Determine an equation in the form of $h(x) = a \cos[b(x - c)] + d$ that defines f as a cosine function.

• Sketch the graph of $y = \left| f(x) \right|$ as it would appear on a graphing calculator with the viewing window $x : \left[0,\ 2\pi,\ \frac{\pi}{2}\right]$ $y : \left[-6,\ 6,\ 1\right]$.



• Determine the roots of $\left| f(x) \right| = 0$, and express your answer as a general solution.

You have now completed the examination.
If you have time, you may wish to check your answers.

Tear-out Page

Fold and tear along perforation.

Pure Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For two points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Graphing Calculator Window Format

x: $[x_{\min},\ x_{\max},\ x_{\text{scl}}]$

y: $[y_{\min},\ y_{\max},\ y_{\text{scl}}]$

Exponents, Logarithms, and Geometric Series

$$\log_a(M \times N) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a M^n = n \log_a M$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S_n = \frac{rt_n - a}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\quad |r| < 1$$

Statistics

$$\mu = np \qquad \sigma = \sqrt{np(1 - p)}$$

$$z = \frac{x - \mu}{\sigma}$$

If $np \geq 5$ **and** $n(1 - p) \geq 5$, then the binomial distribution is a large sample.

Probability

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$P(k) = {}_nC_k\ p^k(1 - p)^{n-k}$$

Permutations and Combinations

$${}_nP_r = \frac{n!}{(n - r)!}$$

$${}_nC_r = \frac{n!}{(n - r)!\,r!}$$

In the expansion of $(x + y)^n$, the general term is $t_{k+1} = {}_nC_k\ x^{n-k}y^k$.

Trigonometry

$$a = r\theta$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

Conics

General Form

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Standard Form

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = +1$$

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = \pm 1$$

$$y - k = a(x - h)^2$$

$$x - h = a(y - k)^2$$

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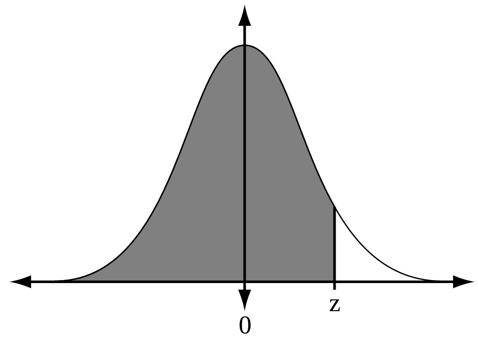
$$z = \frac{x - \mu}{\sigma}$$



Areas under the Standard Normal Curve

z	**0.09**	**0.08**	**0.07**	**0.06**	**0.05**	**0.04**	**0.03**	**0.02**	**0.01**	**0.00**
–3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
–3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
–3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
–3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
–3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
–2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
–2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
–2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
–2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
–0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
–0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
–0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
–0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

Areas under the Standard Normal Curve

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

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Page

No marks will be given for work done on this page.

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Pure Mathematics 30

January 2002

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Pure Mathematics 30 Diploma Examination January, 2002

Multiple-Choice Key, Numerical-Response Key and Sample Answers to Written-Response Questions

PURE MATHEMATICS 30 - January, 2002

MULTIPLE-CHOICE KEY

1.	B	**18.**	D
2.	D	**19.**	D
3.	A	**20.**	A
4.	D	**21.**	B
5.	C	**22.**	D
6.	B	**23.**	C
7.	B	**24.**	A
8.	C	**25.**	C
9.	D	**26.**	B
10.	A	**27.**	D
11.	B	**28.**	A
12.	D	**29.**	B
13.	C	**30.**	A
14.	C	**31.**	A
15.	A	**32.**	C
16.	C	**33.**	B
17.	B		

NUMERICAL-RESPONSE KEY

1.	132
2.	0.27
3.	23.9
4.	32
5.	2
6.	6.7

This scoring guide reflects a mark based on four criteria:

- mathematical understanding
- application of processes
- clarity of communication
- use of technology

	GENERAL SCORING GUIDE
1 mark	In the response, the student • applies some relevant mathematical knowledge to explore the initial stages of the problem; however, the response reflects a misunderstanding of the problem • uses a relevant strategy, mathematical process, or problem-solving technique to explore the initial stages of the problem • communicates very little relevant information and the response lacks clarity • uses technology inappropriately or the use of technology is not evident
2 marks	In the response, the student • applies some relevant mathematical knowledge to find partial solutions to the problem; however, the response reflects a minimal understanding of the problem • uses relevant strategies, mathematical processes, or problem-solving techniques to find a partial solution • communicates strategies in a manner that lacks clarity or is incomplete • uses technology where appropriate; however, errors are evident
3 marks	In the response, the student • applies mathematical knowledge to find partial solutions to the problem and reflects a basic understanding of the problem • uses appropriate strategies, mathematical processes, and problem-solving techniques to find partial solutions to the problem • communicates strategies and solutions in an organized manner; however, errors, inconsistencies, and omissions affect clarity • uses technology appropriately; however, there are inconsistencies in their application
4 marks	In the response, the student • applies appropriate mathematical knowledge to find a complete solution to the problem and reflects a good understanding of the problem • uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete solution to the problem; however, the solution contains an error that hinders understanding of the response • communicates strategies and solutions in an organized manner; however, errors or omissions may affect clarity • uses technology appropriately
5 marks	In the response, the student • applies appropriate mathematical knowledge to find a complete and correct solution to the problem and reflects an excellent understanding of the problem • uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete, correct solution; the solution may have a minor error but it does not hinder the understanding of the response • communicates strategies and solutions in a clear, complete, and organized manner that reflects a thorough understanding of the problem • uses technology effectively

SAMPLE ANSWERS TO THE WRITTEN-RESPONSE SECTION

Note: The responses that follow represent **ONE** approach to each of the problems. During the diploma examination marking session, provision is made for considering the various approaches students may have used.

Sample Answer for Written Response 1—Total: 10%

Written Response—10%

1. Two students, Peter and Ken, performed a simulation that involved the tossing of 3 coins. They recorded the number of heads and tails from 10 trials, as shown below.

Trial 1	HHT	Trial 6	THH
Trial 2	HHT	Trial 7	HTH
Trail 3	HTT	Trial 8	TTT
Trial 4	HHH	Trial 9	HHH
Trial 5	HTT	Trial 10	TTT

- According to the results from the simulation above, what is the experimental probability of getting 3 heads on a single trial?

A POSSIBLE SOLUTION

$P(\text{HHH}) = \frac{2}{10} = 0.20$

Use the following information to answer the next part of the question.

Ken stated that the experimental probability of getting heads in this simulation was higher than he had expected. He defined the sample space as HHH HHT HTH HTT TTT TTH THT THH Ken reasoned that because there were 8 possible outcomes, the probability of getting 3 heads on a single trial was $\frac{1}{8}$.	Peter suggested that the sample space should be 3 heads 2 heads and 1 tail 3 tails 2 tails and 1 head Peter, reasoned that because there were only 4 possible outcomes, the probability of getting 3 heads on a single trial was $\frac{1}{4}$.

- Determine who was correct and justify your choice.

A POSSIBLE SOLUTION

There are eight distinct outcomes where order is important. If order is not important then there are four outcomes but each outcome would have a number of frequencies. Peter does not distinguish between an outcome of the sample space and an event.

3 heads	would occur only once
2 heads and a tail	would occur 3 times
2 tails and a head	would occur 3 times
3 tails	would occur only once

Therefore Ken is correct.

- Use the formula ${}_nC_k\, p^k(1-p)^{n-k}$ and the sample space that **you chose** in the previous bullet to determine, to the nearest thousandth, the theoretical probability of flipping exactly 2 sets of HHH in 10 trials.

A POSSIBLE SOLUTION	
Using Ken's probability	Using Peter's probability
${}_{10}C_2(0.125)^2(0.875)^8 = 0.242$	${}_{10}C_2(0.25)^2(0.75)^8 = 0.282$

- Ken suggests that the probability of flipping exactly 2 sets of HHH in 10 trials can be modelled using a normal approximation of the binomial distribution.
  Is Ken correct? Justify your answer.

A POSSIBLE SOLUTION

No, Ken is not correct.

Using Ken's probability

$$np = 10(0.125) = 1.25 \qquad n(1-p) = 10(0.875) = 8.75$$

(1) Both quantities must be larger than or equal to 5 for a normal approximation to the binomial to take place.

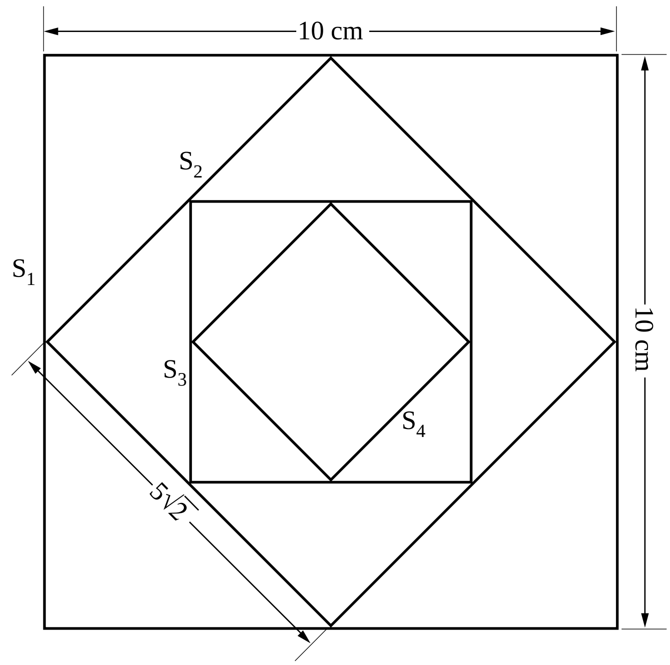
or

(2) Interval length is less than 5.

Sample Answer for Written Response 2—Total: 15%

Use the following information to answer the next question.

A square, S_1, with sides of 10 cm is modelled below. A second square, S_2, with sides of $5\sqrt{2}$ cm is inscribed in S_1 so that the vertices of S_2 lie at the midpoints of the sides of S_1. A third square, S_3, is inscribed in S_2 so that the vertices of S_3 lie at the midpoints of the sides of S_2. This process is continued indefinitely.



Written Response—15%

2. • The lengths of the sides of S_1, S_2, S_3, … form a geometric sequence. Determine the exact length of a side of S_3.

A POSSIBLE SOLUTION

$10,\ 5\sqrt{2}$

$$r = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2} \text{ or } \frac{1}{\sqrt{2}}$$

$$S_3 = 5\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$S_3 = 5 \text{ cm}$$

- The **perimeter** of each square forms a term in a geometric sequence. Determine the **exact** value of the **perimeter** of S_4.

A POSSIBLE SOLUTION
$P_1 = 40,\ P_2 = 20\sqrt{2}$

$$P_4 = 40\left(\frac{1}{2}\sqrt{2}\right)^{4-1}$$

$$= 40\left(\frac{1}{8}\right)\left(2\sqrt{2}\right)$$

$$P_4 = \sqrt{200}$$

$$= 10\sqrt{2}\ \text{cm}$$

or

$$P_4 = 40\left(\frac{1}{\sqrt{2}}\right)^3$$

$$= \frac{40}{\sqrt{8}}$$

$$\text{or } = \frac{20}{\sqrt{2}}\ \text{cm}$$

$$= 10\sqrt{2}\ \text{cm}$$

- If the process of drawing the squares were to continue indefinitely, then what would be the sum of the infinite geometric series of the squares' **perimeter** values, correct to the nearest hundredth of a centimetre?

A POSSIBLE SOLUTION

Infinite Geometric series.

$a = 40,\ r = \frac{\sqrt{2}}{2}$

$$S = \frac{a}{1-r}$$

$$S = \frac{40}{1-\frac{\sqrt{2}}{2}}$$

$$S = \frac{80}{2-\sqrt{2}} \approx 136.57\ \text{cm}$$

- Show how the **areas** of consecutive squares form a sequence that is geometric.

A POSSIBLE SOLUTION
Sequence: $A_1 = 100 \text{ cm}^2$, $A_2 = 50 \text{ cm}^2$, $A_3 = 25 \text{ cm}^2$ $r = \frac{50}{100}$ and $r = \frac{25}{50}$ Therefore geometric sequence with common ratio of $\frac{1}{2}$.

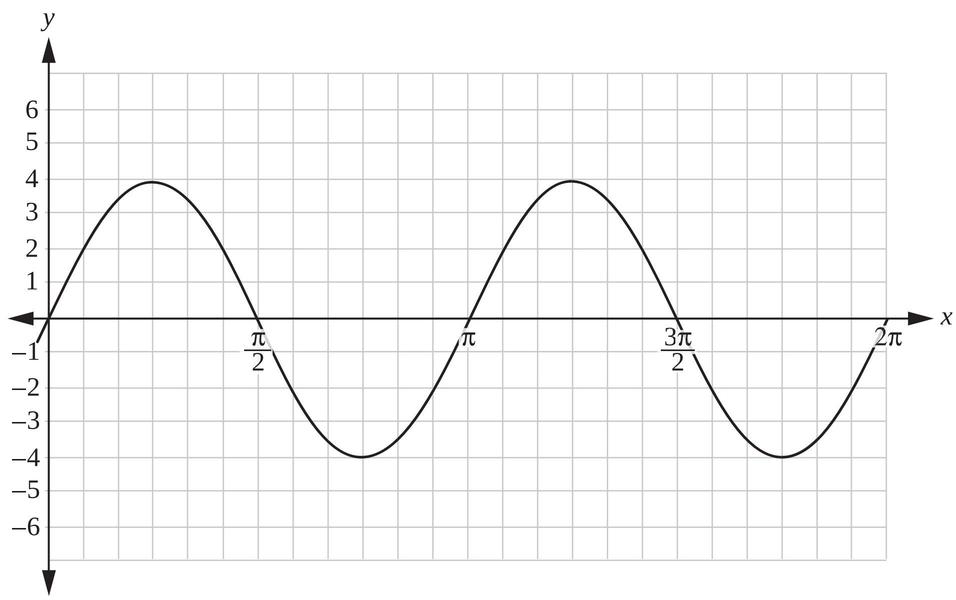
- Determine an expression that represents the sum of the first k terms of the geometric sequence calculated in the previous bullet.

A POSSIBLE SOLUTION
$S_k = \dfrac{100\left[\left(\frac{1}{2}\right)^k - 1\right]}{\frac{1}{2} - 1}$ **or** $S_k = -200\left[\left(\frac{1}{2}\right)^k - 1\right]$ **or** $S_k = 200\left[1 - \left(\frac{1}{2}\right)^k\right]$

Sample Answer for Written Response 3—Total: 10%

Use the following information to answer the next question.

The partial graph of $f(x) = 8 \cos x \sin x$ is shown below.



Written Response—10%

3. • Identify the amplitude and the period of f.

A POSSIBLE SOLUTION	
Amplitude	4
Period	π or 180°

- Determine an equation in the form of $g(x) = a\sin[b(x-c)] + d$ that defines f as a sine function.

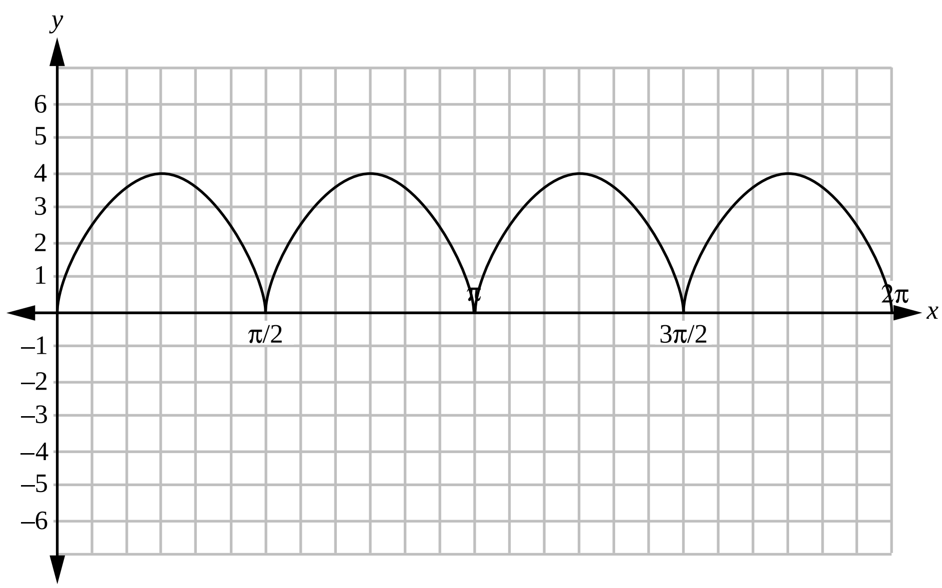
A POSSIBLE SOLUTION
$a = 4$ $b = \frac{2\pi}{\pi} = 2$ $c = 0$ $d = 0$ $y = 4\sin(2x)$ **or** $g(x) = 8\cos x\sin x = 4\cos x\sin x + 4\cos x\sin x$ $= 4\sin(2x)$

- Determine an equation in the form of $h(x) = a\cos[b(x-c)] + d$ that defines f as a cosine function.

A POSSIBLE SOLUTION
Amplitude 4 Period $\pi \therefore b = 2$ Cos x shifted right $\frac{\pi}{4}$, $\therefore c = \frac{\pi}{4}$ or 45^{o} No vertical displacement, $\therefore d = 0$ $h(x) = 4\cos\left[2\left(x - \frac{\pi}{4}\right)\right]$ **or** $h(x) = 4\cos[2(x - 45°)]$

- Sketch the graph of $y = \left| f(x) \right|$ as it would appear on a graphing calculator with the viewing window $x : \left[0,\ 2\pi,\ \frac{\pi}{2}\right]$ $y : \left[-6,\ 6,\ 1\right]$.

A POSSIBLE SOLUTION



- Determine the roots of $\left| f(x) \right| = 0$, and express your answer as a general solution.

A POSSIBLE SOLUTION

$|f(x)| = 0,\ x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ **and** $x = n\frac{\pi}{2},\ n \in I^*$ **or** $x = 0 \pm \frac{n\pi}{2},\ n \in I^*$

or $x = \pm\frac{n\pi}{2},\ n \in W$

* *Z* can also be used to represent integers