Pure Mathematics 30

Teacher Notes: Skydiving

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Alberta Education

Pure Mathematics 30

Skydiving—Teacher Notes

Introduction

This project relates to some of the mathematics involved in describing falling objects, sometimes ignoring air resistance and sometimes including the effects of air resistance. It allows students to use the knowledge they have gained from the Conic Sections unit, the Exponents, Logarithms, and Geometric Series unit, as well as from the Statistics unit. The project is designed to be completed in three to five hours of student time. The use of this project is optional; however, if you do choose to use it, you may include it as part of your assessment. Sample solutions for the project questions can be found on the Alberta Education extranet at https://phoenix.edc.gov.ab.ca. A hard copy of the solutions will be mailed to your school in late January 2009. The general scoring guide for the project is the same as the one issued in September 2000.

The first written-response question on the Pure Mathematics 30 June 2009 diploma examination will be related to this project. This question is worth 10% of the total mark on the diploma examination. Students who do not complete the project but who have completed the course will have the knowledge to answer the written-response question; however, students who do complete the project will gain experience with the related mathematical skills.

Specific Notes

Teachers may want to

- remind students that the same parabolic path can have different standard-form equations, depending on the location of the origin of the coordinate plane (part A, questions 1 and 2)
- discuss with students the need to change models if predictions differ greatly from experimental values (part A, question 3)
- recognize the need for units for the coefficient k, but not expect students to know or state the units for k (part B, questions 1 and 2, and part C, questions 1, 2, 3, and 4)
- show that exponential expressions like $y = 1 - b^t$ have limits as t becomes very large (part B, question 3)
- encourage their Mathematics 31 students to solve the differential equation $v'(t) = g - kv$ to obtain the expression $v(t) = \frac{g}{k}\left(1 - e^{-kt}\right)$ (part B, questions 2 and 3)
- remind students that not all quantities are normally distributed (part C, questions 2, 3, and 4)

Program of Studies

The project relates to mathematics learned in the following units of Pure Mathematics 30.

Exponents, Logarithms, and Geometric Series

Specific Outcomes

2.4: Use the laws of exponents and logarithms to:
- solve and verify exponential equations and identities
- solve logarithmic equations
- simplify logarithmic expressions.

[R]

2.6: Model, graph and apply exponential functions to solve problems. [PS, T, V]

Conic Sections

Specific Outcomes

4.2: Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only) [CN, T, V]

4.3: Convert a given equation of a conic section from general to standard form and vice versa. [R, T]

Statistics

Specific Outcome

6.3: Use z-scores to solve problems related to the normal distribution. [PS, R, T, V]

ICT Program of Studies

C.6—Students will use technology to investigate and/or solve problems.

Specific Outcomes

4.1: Investigate and solve problems of prediction, calculation, and inference.

4.2 Investigate and solve problems of organization and manipulation of information.

4.3 Manipulate data by using charting and graphing technologies in order to test inferences and probabilities.

F.1—Students will demonstrate an understanding of the nature of technology.

Specific Outcome

4.2: Solve mathematical and scientific problems by selecting appropriate technology to perform calculations and experiments.

P.2—Students will organize and manipulate data.

Specific Outcome

4.1: Manipulate and present data through the selection of appropriate tools, such as scientific instrumentation, calculators, databases, and/or spreadsheets.

Mathematical Processes

The seven mathematical processes identified in the Program of Studies are addressed in this project in the following manner.

Communication	Describe how to relate the design coefficient *k* and the shape of an object to the terminal velocity of the object when it falls.
Connections	Find and use the connection between design coefficient *k* and terminal velocity for a falling object.
Estimation and Mental Mathematics	Check the reasonableness of calculator and spreadsheet solutions.
Problem Solving	Determine velocities and times for various falling objects. Determine probabilities for terminal velocities when terminal velocities are not normally distributed.
Reasoning	Decide when the effects of air resistance have to be considered for different falling objects.
Technology	Use a calculator and/or spreadsheet to enter lists and to graph scatter plots from data. Generate exponential functions. Use graphing techniques to estimate solutions.
Visualization	Visualize the terminal velocities and the times to reach terminal velocity for different shaped objects. Visualize how the same parabolic path can have different equations if the origin of the coordinate plane is changed.