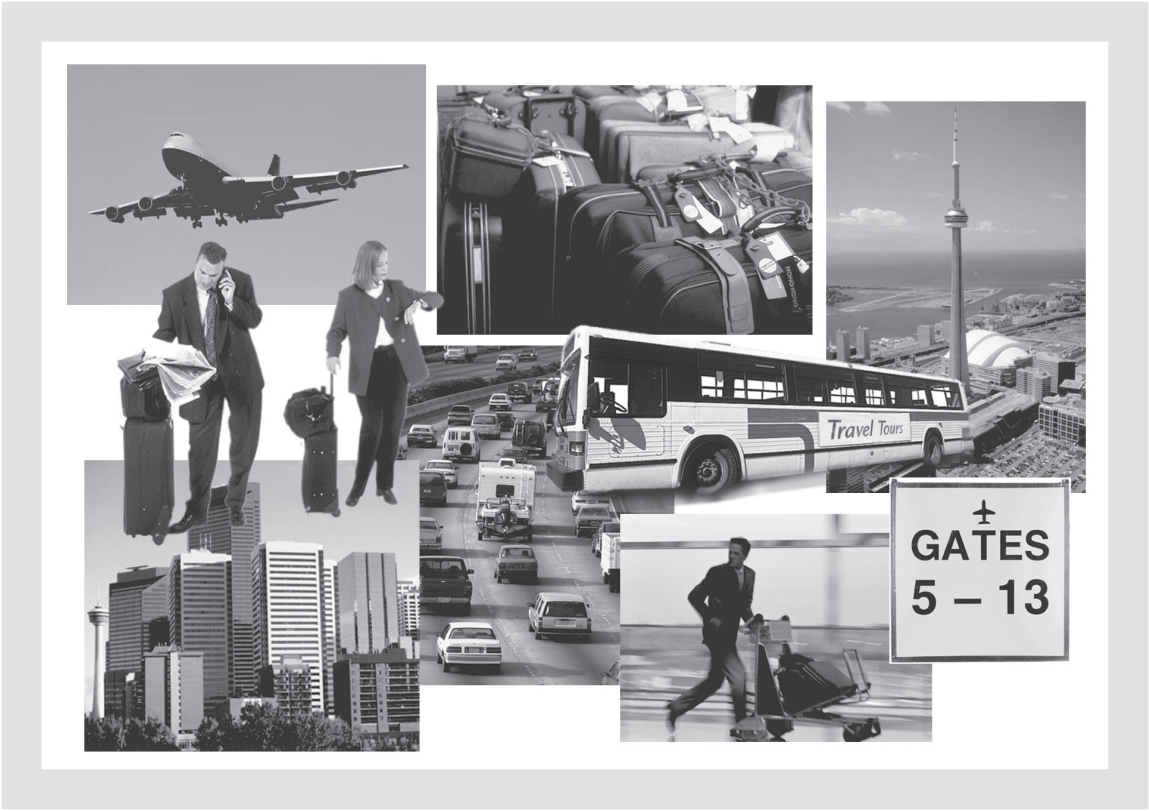
Applied Mathematics 30

Student Project: Conference Travel



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Project—Conference Travel

Introduction

Many transportation companies overbook their scheduled routes in order to maximize their revenue. This project will simulate the practice of overbooking and help you to understand some of the reasons companies do it.

A mathematics teacher from southern Alberta is going to attend an educators' convention in Toronto, Ontario. She books a flight from Lethbridge to Toronto via Calgary and pre-purchases a ticket on a shuttle bus from the Toronto airport to the convention hotel.

Part A

For the flight from Calgary to Toronto, the airline company uses an airplane with a seating capacity of 133 passengers. The airline company knows that, on average, only 90% of passengers show up for any given flight. To compensate for the "no-shows," the airline company overbooks each flight, hoping to fill the airplane with as close to 133 passengers as possible. If, however, more passengers show up for a flight than there are seats available, some passengers will be "bumped" to the next scheduled flight.

To determine the possible number of passengers who will show up for their flights, the airline company conducted 100 simulations. The results of the simulations giving the number of passengers that showed up for a flight are shown below.

Number of passengers	123	124	125	126	127	128	129	130	131	132	133
Frequency	1	2	0	0	6	4	4	8	12	10	8

Number of passengers	134	135	136	137	138	139	140	141	142
Frequency	14	10	6	5	6	1	2	1	0

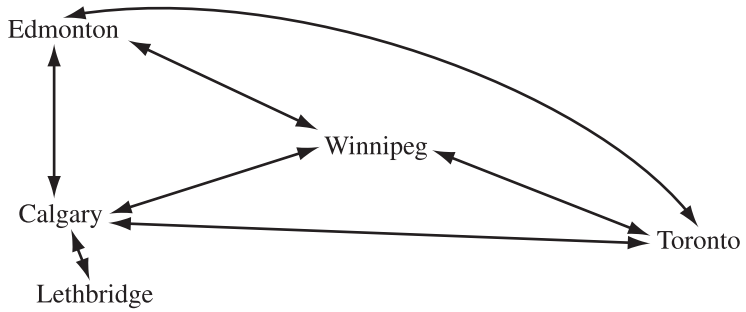
1.
- Using the results of the simulations, determine the probability that, at most, 133 passengers will show up for a flight.
- What is the probability that one or more passengers will be bumped?
- The airline company wants to predict how many of the next 50 flights from Calgary to Toronto will have more passengers show up than there are seats available. Determine the symmetric 95% confidence interval for the number of flights from which at least one passenger will be bumped.

2. The airline company determined that the frequency of the number of passengers who show up for a flight approximates a normal distribution.

- Using the simulation data, determine the mean number of passengers who show up for a flight, to the nearest whole number, and the standard deviation, to the nearest tenth.

- If 147 tickets are sold, and the plane can only accommodate 133 passengers, what is the probability that at least one passenger will be bumped?

3. The teacher knows that she might be bumped because airlines overbook. Before she leaves home, she draws a network diagram of the airline's flight routes from Lethbridge to Toronto. The network is shown below.



- Determine the number of routes a passenger could fly from Lethbridge to Toronto.

- Construct a matrix, A, that represents the number of direct flights between the cities on this network diagram. Use a 1 to represent a direct flight from one city to another; use a 0 when a flight between two cities is not available.

- Determine the matrix A^2. What does this matrix represent?

- The teacher wants to fly from Lethbridge to Toronto and have at most two stopovers. How many different routes are possible? Justify your answer.

Part B

The shuttle bus company, Shuttleship, transports airline passengers between the airport and the passengers' hotels. A bus holds 9 passengers at a time. Tickets to ride the bus are sold in advance. On average, 10% of those who pre-purchase tickets do not show up for their ride. To compensate for this, Shuttleship sells 10 tickets for each one-way trip. Occasionally, all 10 people show up, and one person needs to be bumped from the bus.

The teacher is arriving in Toronto on a day when many of the other convention delegates are also arriving. Shuttleship has pre-sold 10 tickets on each of its 50 one-way trips from the airport to the convention hotel. The table below shows a summary of the 50 one-way trips made that day.

Number of No-Shows	Tally	Frequency
0 (1 customer bumped)	𝍸 𝍸 𝍸 ||||	19
1 (no customers bumped)	𝍸 𝍸 𝍸	
2 (1 empty seat)	𝍸 ||||	
3 (2 empty seats)	𝍸 ||	

1. • Complete the frequency column in the above table.

• Using the data above, determine the expected number of times, out of 50, that Shuttleship would have 0 no-shows.

2. • Construct a histogram for these data. Can these data be modelled by a normal distribution? Explain your answer.

• Write a regression equation that can be used to model these data. Justify your choice of equation mathematically.

3. Shuttleship's profit on a one-way trip is based on the number of occupied seats. When Shuttleship has to turn away a paid passenger, it must pay his or her taxi fare, which averages $20. The table below shows the company's profit on a one-way trip. The $20 taxi fare is reflected in the profit shown when a customer is bumped.

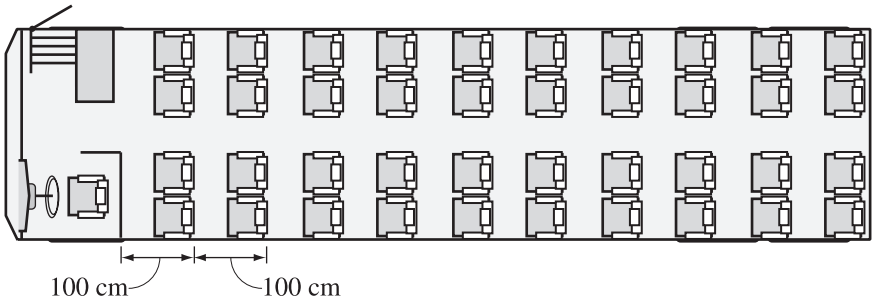
	Profit
A full bus with a bumped customer	$10
A full bus without a bumped customer	$30
One empty seat	$15
Two or more empty seats	$0

- Design a spreadsheet that Shuttleship could use to calculate the expected profit for the day. Supply two printouts—one printout showing the results of the calculations and the other printout showing the formulas in the cells.

Part C

Shuttleship also offers nighttime bus tours of Toronto. It uses a larger, more luxurious bus for these tours.

1. Currently, the tour bus has 10 rows, and each row has 4 seats. Each row takes up 100 cm of length, as shown in the diagram below.



- Shuttleship plans to renovate this bus. It will add 20 cm to each row for extra leg room and re-cover the seats at a cost of $468/seat, which includes all applicable taxes. How much will it cost to re-cover the seats that will be used on the renovated bus?

- Before the upgrade, Shuttleship generated $2 000 in revenue when all 40 seats were sold. To maintain this same amount of revenue with fewer seats, by how much will Shutteship have to increase the current ticket price?

Part D

1. Research the reasons airlines overbook their flights, and briefly discuss your findings.

2. What type of compensation do passengers receive when they are bumped from a flight?

3. Name another industry that follows the same practice, and explain why it may benefit the industry and what problems it may cause.

The following websites may help in your research:

www.ttgweb.com/cms/11.html
www.cta-otc.gc.ca

Note: Website addresses sometimes change. If the websites listed above are not available, use a search engine and type in keywords such as those listed below:

"overbooking"